Université Paris-Dauphine

Exam Python for Finance

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This is an open book exam. You are allowed to use any course material and any other information on the internet.

At the end of the exam, send your solutions to `andras.niedermayer@dauphine.fr` as a Jupyter notebook containing the solutions. Please also specify whether you used Python 2 or Python 3.

1. Download the data "Data for Lecture 5 – zip file" from `http://andras.niedermayer.ch/teaching/` and unzip the files into your working directory.
   - Plot a histogram of the transaction prices of Volvo in the data set.
   - How close is the distribution of Volvo prices to a normal distribution? Use a quantile-quantile plot to illustrate this.
   - How close is the distribution of Volvo prices to a *log*-normal distribution? Use a quantile-quantile plot to illustrate this.

2. Take a portfolio optimization problem with three assets with expected returns $\mu = (1, 1, 1)$ and covariance matrix

$$\Sigma = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}.$$

An investor chooses portfolio weights w to maximize the objective function $f(w) = w'\mu - \gamma w'\Sigma w$ for some $\gamma \geq 0$.

- Is it true that the investor will never buy asset 3, since it has the largest variance? Why or why not? Illustrate this with two examples of portfolios for $\gamma = 0.5$.
- Generate 1000 random portfolios (i.e. random weights) and plot a graph with the volatilities and returns of these portfolios.
- Plot the distribution of the value of the objective function for these random portfolios for $\gamma = 0.5$.
- Maximize the objective function for $\gamma = 0.5$ and the constraints $w_i \in [0, 1]$ and $\sum_i w_i = 1$. What is the optimal weight? What is the return and volatility of this portfolio?
- Maximize the objective function for $\gamma = 1$ and the constraints $w_i \in [0, 1]$ and $\sum_i w_i = 1$. Add the additional constraint that no asset can have a weight larger than 35%. What is the optimal weight? What is the return and volatility of this portfolio?

3. Do a Monte Carlo simulation of the Black-Scholes model

$$dS_t = r \times dt + \sigma \times dz_t$$

with $S_0 = 100$, $r = 0.1$ and $\sigma = 0.5$. Consider two years. Simulate 10,000 possible stock paths, take 25 steps per year.

- Plot a histogram of the distribution of stock prices at $T = 1$ and at $T = 2$ based on the possible stock paths.
- Based on this simulation, compute the value of a European call option with strike price 100 and date $T = 2$.
- Compute the value at risk (computed at $T = 1$) with a probability of 0.1% (that is, $\alpha = 0.001$) based on the simulated path.