Industrial Economics

Asymmetries of Information

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The basis of insurance: risk aversion

- The theory of probability and the theoretical treatment of risk have their historical roots in considerations about gambling.
- A particular game was proposed by the Basel mathematician Bernoulli in order to demonstrate risk aversion.
- This game became famous under the name of the "St. Petersburg paradox", where it was first proposed.

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- Bernoulli asked people the price they would pay for the right to participate in a particular bet that offered potentially unlimited gains.
- The proposed game was a simple variation of "heads or tails".
- Say, the participant selects "heads".
- A coin is then tossed as many times as it is required for "heads" to turn up.
- Then the participant receives payment.

- The peculiarity of the game consists in determining the amount of payment.
- If head comes up in the first round, the participant receives only 2 € (equal to 2^1),
- if head comes up in the second round he will receive $4\,€ = 2^2$,
- if it come up in the *n*th round, he will receive 2^n.

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- With a bit of luck, the gains can become quite large. In theory the expected gain is even unlimited with

$$\text{total expected gain} = \frac{1}{2}\times 2^1 + \frac{1}{4}\times 2^2 + \frac{1}{8}\times 2^3 + \ldots = \sum_{n=1}^{\infty} \frac{1}{2^n}\times 2^n = \infty$$

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- Even in the worst case, the player will receive EUR 2.
- Bernoulli now asked "How much would you offer in order to be allowed to play this game with me?"
- The sums on offer were quite modest. This is the paradox (see `https://en.wikipedia.org/wiki/St._Petersburg_paradox#Expected_utility_theory`).
- The solutions on offer are two:
  1. Risk aversion or declining marginal utility of income: a certain loss, the price of entry, weighs more heavily than an uncertain gain, even if the latter is theoretically higher.
  2. Even a risk neutral person would only pay relatively modest amounts, if the bank's resources were limited (which in reality, of course, they are).
     - That means gains are never infinite.
     - Even if the casino's resources were EUR 1 billion, the value of the lottery would be only EUR 31!

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- We are here interested in risk aversion, where the certainty of an average income, i.e., not participating in the bet, is preferred to the expected utility of doubling or losing that income.
- The graph below shows the corresponding utility curve.
- $\rightarrow$ draw graph here
- A risk averse individual will always prefer to receive a stable income of EUR 30 to an arrangement where he received half the time EUR 10 and the other half of the time EUR 50.
- Expressed formally this yields

$$\underbrace{Utility(30)}_{\text{Case A}} > \underbrace{\frac{1}{2}Utility(10) + \frac{1}{2}Utility(50)}_{\text{Case B}}$$

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- Loss aversion is a third term covering the same phenomenon.
- The gain in the utility due to an additional unit of income is less than the loss of utility when losing the same amount.
- This form of preferences also explains insurance.
- Insurance guarantees a stable income.

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- Imagine the following:
  - a house with a value EUR 40 that can be insured against fire,
  - the probability of a fire during the lifetime of the house is $\frac{1}{2}$ and
  - that the cost of insurance is hence EUR 20.
- If ensured, the owner will have a constant income of EUR 30 (Case A).
- If the owner does not take insurance, he will have
  - with a probability of $\frac{1}{2}$ an income of EUR 50 and
  - with a probability of $\frac{1}{2}$ an income of EUR 10.
- If the owner is risk averse he will prefer to insure his house.

- The graph provides further interesting information.
- Insurance firms aim at maximizing their profits.
- Insuring homes with a EUR 40 value that burn with a probability of $\frac{1}{2}$ for a price of EUR 20 will not allow making any profit given the unavoidable transaction costs.
- They will thus take a premium.

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- This premium ΔI in the graph above corresponds to the difference between a stable income and the expected pay-off of risk taking, both yielding the same utility for the customer.
- In other words, if the insurance is capable of capturing for itself the complete benefits of insurance to the customer, its profits will be ΔI.
- As in the case of the franchise, it is probably wise to leave a portion of this surplus to the customer in order to internalise any negative externalities.

The impact of informational asymmetries

- George Akerlof showed in his famous paper "The Market for Lemons" how asymmetric information leading to
  - suboptimal equilibria
  - and a restriction
  - or even disappearance of the market.
- Markets with asymmetric information concern
  - used cars,
  - insurance,
  - restaurants etc.
  - or any good whose value can only be observed after its acquisition.

The impact of informational asymmetries

- In “Equilibrium in Competitive Insurance Markets”, Rothschild and Stiglitz show how well-structured contracts can at least create submarkets to overcoming informational asymmetries.
- The market for car insurance is a good example.
- The insurer cannot distinguish between high risk- and low risk-drivers.
- Selling policies at average cost (average cost of accident times average probability) would cause losses, since the high-risk drivers buy more insurance than low-risk drivers.

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- If q indicates the quality of second-hand cars, where q is a random variable distributed uniformly on the interval [0, 1],
- then the average quality of a car is the expected value of q, i.e., $1/2$.
- The market consists of a large number of buyers and sellers.
- The former have a reserve price (utility) equal to $3/2*q$ for vehicle of quality q,
- while sellers want to at least obtain q.

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- In principle, there is thus ample space for gains of trade.
- It is assumed that buyers are risk neutral at the risk and ignorant of the quality of the vehicle in front of them.
- They know that q is uniformly distributed.
- Buyers thus calculates the expected quality with $q^e = \frac{1}{2}$.
- They are therefore willing to pay $3/2 \times 1/2 = \frac{3}{4}$ for the unknown car in front of them.

- If the quality of each vehicle was observable, each seller would sell his vehicle at a price at least q, since the buyers are willing to pay 3/2*q,
- $\Rightarrow$ each car would be sold at a price between q and 3/2*q.
- However, given that buyers offer $\frac{3}{4}$ regardless of the quality if the car sellers with vehicles with $q > \frac{3}{4}$ will leave the market.
- The quality of the remaining vehicles then lies between 0 and $\frac{3}{4}$.

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- Buyers know this and adjust to $q^e = 3/8$.
- Henceforth, buyers will only be willing to pay $3/2*3/8 = 9/16$.
- $\Rightarrow$ withdrawal of the sellers of cars whose quality is higher than $9/16$.
- By repeating this reasoning, it is easy to see that in the end remains only a small market for vehicles with q $= 0$, which are basically given away.

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- The paper by Rothschild and Stiglitz shows how insurers protect against informational asymmetry and market disappearance.
- They do this by using a mechanism of self-selection in a separating equilibrium.
- The idea is to offer two contracts that are structured in a way that high risk drivers prefer the specific contract for high risks.
- This is achieved by reducing the amount of coverage of the contract offered to low risk (franchise).
- Insurers and high risk drivers maintain their utility in this arrangement.
- However, low risk drivers will only be able to obtain partial insurance.

The model

- There are two types of drivers with different probabilities of accident risk,
  - p_L (low risk) and
  - p_H (high risk).
- This is about adverse selection, not moral hazard.
- The initial wealth is W, and becomes either
  - W_1 (without an accident) or
  - W_2 (with an accident)
- Damages are D
- The probability of an accident is p.
- The insurance premium is α.
- The compensation received from the insurance in case of an accident is $\hat{a}$.

- It thus holds that:

	No accident	Accident
With insurance	$W_1 = W - \alpha$	$W_2 = W - D - \alpha + \hat{a}$
Without insurance	$W_1 = W$	$W_2 = W - D$

- The expected utility of an uninsured agent is thus:

$$\hat{U}_{SA} = (1-p)U(W) + pU(W-D).$$

- The expected utility of an insured agent instead is:

$$\hat{U}_{SA} = (1-p)U(W-\alpha) + pU(W-D-\alpha+\hat{a}).$$

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- The model is driven by two concerns:
  1. The agents would like to smooth their utility in the two states W_1 and W_2 as they are risk averse.
  2. Insurers will need to make at least zero profits or $\Pi_I = \alpha - p\hat{a} \geq 0$.
     - With perfect competition it will hold that $\Pi = \alpha - p\hat{a} = 0$ or $\alpha = p\hat{a}$.

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- $\rightarrow$ draw figure here
- The slope of the line from point E (for “endowment” or initial wealth) is called the “fair odds-line”.
- All along this line it holds under competition that $\alpha = p\hat{a}$ or $\alpha = pD$ with full insurance.
- Points below that line imply a gain for the insurer, points above that line it implies a loss.

- The slope of the fair odds-line (FOL) line is

$$Slope_{FOL} = -\frac{D - \alpha}{\alpha}.$$

Under competition with $\alpha = pD$ this becomes:

$$Slope_{FOL} = -\frac{D - pD}{pD} = -\frac{1 - p}{p}.$$

- This implies the higher the probability of an accident, the more horizontal will be the fair odds-line.
- Due to risk aversion, agents seeking insurance will always seek out the tangency point between their indifference curves and the fair odds-line.

- In the next case, the graph shows two different groups of agents seeking insurance, who have respectively accident probabilities that are either high (p_H) or low (p_L).
- $\rightarrow$ draw figure here
- Under perfect information we obtain two equilibrium points for each of the groups.
- With asymmetric information, the problem is that high risks will buy inexpensive insurance claiming that they are low risks.
- In the graph, it is visible that $\alpha_H > \alpha_L$ and that the indifference curve of the low risks is higher.
- Insurers however would go bankrupt as $\alpha_L < \hat{a}p_H$ assuming full insurance.

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- $\rightarrow$ add figure here
- Insurers want to protect themselves against asymmetric information and the disappearance of the market.
- For this purpose, they offer two contracts that will induce the two groups of customers to self-select.
- One offer will be the old contract for the high-risks, offering full protection but at a very high premium with $\alpha_H = p_H \hat{a}$.
- The other offer will be a new contract for the low risks which only partially covers damages only partially with $\alpha_L = p_L \hat{a}_L$.

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- This separating equilibrium (H/S) will force the high risks to choose their old contract as it would allow them a slightly higher utility
- (partial insurance is of no interest to them).
- Low risk customers will choose S as L is no longer on offer.

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- Compared to the full information equilibrium, low risk customers are considerably worse off, as they can only have partial insurance.
- Choosing H would mean vastly overpaying for insurance.
- The difference between equilibrium L and equilibrium S can be represented by the notion of a franchise (F) which corresponds to the distance between
  - the net insurance desired by the low risks $(D - \alpha)$ and
  - the amount offered by the insurance (D_S).
- It therefore holds that $F = (D - \alpha) - D_S$.

Numerical example for calculating utilities in a separating equilibrium of two groups of agents with different accident risk

- The agents p_L have an accident probability $7/15$;
- The agents p_H have an accident probability $4/5$;
- The cost of insurance is α_H for p_H and α_L for p_L.
- Accident compensation is $\hat{a}_i$ for $i = L, H$.
- Damage in case of an accident is $D = 15$.
- Both groups have an initial wealth of $W = 16$.

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- Agents have the utility function $U(W) = \sqrt{W}$
- There is decreasing utility of income for each state, which expresses risk aversion.

- Their payoffs will be

	No accident	Accident
With insurance	$W_1 = 16 - \alpha_i$	$W_2 = 16 - 15 - \alpha_i + \hat{a}_i$
Without insurance	$W_1 = 16$	$W_2 = 16 - 15$

- The agents maximize their expected utilities according to the function:
- low probability agents

$$U_L = \underbrace{(1 - p_L)\sqrt{W_1}}_{\text{without accident}} + \underbrace{p_L\sqrt{W_2}}_{\text{with accident}}$$

- high probability agents

$$U_H = \underbrace{(1 - p_H)\sqrt{W_1}}_{\text{without accident}} + \underbrace{p_H\sqrt{W_2}}_{\text{with accident}} .$$

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- A. With no insurance:

$$U_L^{\text{no}} = (1-p_L)\sqrt{W_1} + p_L\sqrt{W_2} = \frac{8}{15}\sqrt{16} + \frac{7}{15}\sqrt{1} = \frac{13}{5} = 2.6$$

$$U_H^{\text{no}} = (1-p_H)\sqrt{W_1} + p_H\sqrt{W_2} = \frac{1}{5}\sqrt{16} + \frac{4}{5}\sqrt{1} = \frac{8}{5} = 1.6$$

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- B. With insurance and symmetric information (full insurance)

$$\begin{aligned} U_L^{\text{full}} &= (1-p_L)\sqrt{W_1} + p_L\sqrt{W_2} \\ &= \frac{8}{15}\sqrt{W-\alpha_L} + \frac{7}{15}\sqrt{W-D-\alpha_L+\hat{a}_L} \end{aligned}$$

$$\begin{aligned} U_H^{\text{full}} =& (1-p_H)\sqrt{W_1} + p_H\sqrt{W_2} \\ &= \frac{1}{5}\sqrt{W-\alpha_H} + \frac{4}{5}\sqrt{W-D-\alpha_H+\hat{a}_H} \end{aligned}$$

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- Next, let's calculate $\hat{a}_i$ (damage compensation in case of accidents)
- Agents are risk averse.
- $\Rightarrow$ In equilibrium they would like to buy the following amounts of insurance:

$$W_1 = W_2 \rightarrow W - \alpha_L = W - D - \alpha_L + \hat{a}_L \rightarrow \hat{a}_L = D = 15$$
$$\rightarrow W - \alpha_H = W - D - \alpha_H + \hat{a}_H \rightarrow \hat{a}_H = D = 15$$

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- Next, let's calculate α_L and α_H, the insurance premiums for low and high risks.
- We know from the zero profit condition for insurers that it holds that

$$\Pi_L = \alpha_L - p_L D = 0 \rightarrow \alpha_L = p_L D = \frac{7}{15} \times 15 = 7$$
$$\Pi_H = \alpha_H - p_H D = 0 \rightarrow \alpha_H = p_H D = \frac{4}{5} \times 15 = 12.$$

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- Substituting α_L, α_H, $\hat{a}_L$ and $\hat{a}_H$ in the utility function we obtain for the utility with full insurance

$$\begin{aligned} U_L^{\text{full}} &= \frac{8}{15}\sqrt{W-\alpha_L} + \frac{7}{15}\sqrt{W-D-\alpha_L+\hat{a}_L} \\ &= \frac{8}{15}\sqrt{16-7} + \frac{7}{15}\sqrt{16-15-7+15} \\ &= \sqrt{9} = 3 > 2.6; \\ U_H^{\text{full}} &= \frac{1}{5}\sqrt{W-\alpha_H} + \frac{4}{5}\sqrt{W-D-\alpha_H+\hat{a}_H} \\ &= \frac{1}{5}\sqrt{16-12} + \frac{4}{5}\sqrt{16-15-12+15} \\ &= \sqrt{4} = 2 > 1.6. \end{aligned}$$

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- In both cases it is profitable to conclude an insurance policy that guarantees the profitability of insurance ($\Pi \geq 0$).
- The utility increase due to insurance $U_i^{\text{full}} - U_i^{\text{no}}$ happens to be the same for both types of agents, $i = H, L$.
- However, this is a coincidence: changing p_H or p_L would change this.

- C. Under asymmetric information
- The problem is that customers with p_H prefer to buy the insurance of customers with p_L.
- This would give them a higher level of utility:

$$U_H^L = \frac{1}{5}\sqrt{16-7} + \frac{4}{5}\sqrt{16-15-7+16}$$
$$= \sqrt{9} = 3 > 2.$$

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- This would however be unprofitable for insurers:

$$\Pi_H^L = \alpha_L - p_H D = 7 - \frac{4}{5} \times 15 = 7 - 12 = -5 < 0.$$

- Insurers do not have any way of distinguishing between p_L and p_H.
- $\Rightarrow$ Insurers would withdraw from the market and no insurance at all would be available.

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- D. The solution: Separating Equilibrium with self-selection
- Insurers will offer a contract to p_L with a lower level of utility for the p_H than their initial contract ($U_H = 2$).
- Thus the p_H will choose their initially offered contract U_H originally proposed.
- The contract U_L^S (i.e., the new contract offered to low risk, now less advantageous in terms of risk coverage, allowing to establish a separating equilibrium) must still always satisfy the requirement that insurers make no losses.
- It there for holds that:

$$\alpha_L = p_L \hat{a}_L = \frac{7}{15} \hat{a}_L$$

and thus

$$\hat{a}_L = \frac{15}{7} \alpha_L$$

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- The premium (α_L) of the new contract for the L types has to make sure that the H types do not take this contract:

$$U_H^{\text{full}} \geq U_L^S(H)$$

where $U_L^S(H)$ is the utility of type H when taking the separating contract meant for type L.

- Assume that equality holds (or that the left hand side is an epsilon smaller than the right hand side and epsilon is negligible).
- To determine α_L, the price or premium of the new contract it thus holds that

$$\begin{aligned} 2 = U_H^{\text{full}} = U_L^S(H) &= (1 - p_H)\sqrt{W - \alpha_L} + p_H\sqrt{W - D - \alpha_L + \hat{a}_L} \\ &= \frac{1}{5}\sqrt{16 - \alpha_L} + \frac{4}{5}\sqrt{16 - 15 - \alpha_L + \frac{15}{7}\alpha_L} \end{aligned}$$

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- Rearranging to get rid of one of the square roots:

$$
\begin{aligned}
2 &= \frac{1}{5}\sqrt{16-\alpha_L} + \frac{4}{5}\sqrt{1+\frac{8}{7}\alpha_L} \\
10 - 4\sqrt{1+\frac{8}{7}\alpha_L} &= \sqrt{16-\alpha_L} \\
\left(10 - 4\sqrt{1+\frac{8}{7}\alpha_L}\right)^2 &= 16-\alpha_L
\end{aligned}
$$

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- Rearranging to get rid of the other square root:

$$\left(10 - 4\sqrt{1 + \frac{8}{7}\alpha_L}\right)^2 = 16 - \alpha_L$$

$$100 - 80\sqrt{1 + \frac{8}{7}\alpha_L} + 16\left(1 + \frac{8}{7}\alpha_L\right) = 16 - \alpha_L$$

$$80\sqrt{1 + \frac{8}{7}\alpha_L} = 100 + \frac{135}{7}\alpha_L$$

$$6400\left(1 + \frac{8}{7}\alpha_L\right) = \left(100 + \frac{135}{7}\alpha_L\right)^2$$

- $\rightarrow$ quadratic equation for α_L

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- This gives two possible solutions for the premium α_L of the contract offered to the low risks:
  - $\alpha_{L,1} = \frac{28}{729}\left(121 - 4\sqrt{505}\right) \approx 1.19$: correct amount
  - $\alpha_{L,2} = \frac{28}{729}\left(121 + 4\sqrt{505}\right) \approx 8.10$: this would yield a damage compensation claim of $\hat{a}_L = \frac{15}{7}\alpha_{L,2} \approx 17.36 > 15$.
  - This is higher than the real damages.
- Insurers will thus only offer a slightly lower amount at, say, $\hat{a}_L = \frac{15}{7} \times 1.19 - \epsilon = 2.55 - \epsilon << D = 15$, which is far lower than the real damages.

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- The difference is particularly large due to the shape of the utility function.
- In the case of an accident, there is now a franchise (i.e., an uninsured amount) of 15-1.19=13.81 to be paid.
- The low risks p_L therefore obtain in this separating equilibrium only very partial coverage.

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- In this new contract the p_L would obtain the following utility:

$$U_L^S(L) = \frac{8}{15}\sqrt{W - \alpha_{L,1}} + \frac{7}{15}\sqrt{W - D - \alpha_{L,1} + \frac{15}{7}\alpha_{L,1}}$$

which is equal to $\frac{1}{81}\left(5\sqrt{505} + 112\right) \approx 2.77$

- The the p_H would obtain the following utility:

$$U_L^S(H) = \frac{1}{5}\sqrt{W - \alpha_{L,1}} + \frac{4}{5}\sqrt{W - D - \alpha_{L,1} + \frac{15}{7}\alpha_{L,1}} - \epsilon$$

which is equal to $\sqrt{4} - \epsilon = 2 - \epsilon < 2$.

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- The H type can do better with full coverage at $\alpha_H = 12$ and $\hat{a}_H = D = 15$ and $U_H = 2$.
- They will thus stay on their original contract corresponding to the symmetric information situation.
- In choosing it, they will reveal themselves as p_H.
- The p_L lose utility, as they obtained in the full insurance case a utility of $U_L^{\text{full}} = 3 > 2.77 = U_L^S(L)$.
- Nevertheless, their utility at 2.77 remains higher than the one they would have obtained in the case without insurance at $U_L^{\text{no}} = 2.6$, since they have obtained at least partial insurance.