Industrial Economics

Asymmetries of Information

Andras Niedermayer

DAUPHINE
UNIVERSITÉ PARIS

The basis of insurance: risk aversion

- The theory of probability and the theoretical treatment of risk have their historical roots in considerations about gambling.
- A particular game was proposed by the Basel mathematician Bernoulli in order to demonstrate risk aversion.
- This game became famous under the name of the "St. Petersburg paradox", where it was first proposed.

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- Bernoulli asked people the price they would pay for the right to participate in a particular bet that offered potentially unlimited gains.
- The proposed game was a simple variation of "heads or tails".
- Say, the participant selects "heads".
- A coin is then tossed as many times as it is required for "heads" to turn up.
- Then the participant receives payment.

- The peculiarity of the game consists in determining the amount of payment.
- If head comes up in the first round, the participant receives only 2 € (equal to 2^1),
- if head comes up in the second round he will receive $4\,€ = 2^2$,
- if it come up in the *n*th round, he will receive 2^n.

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- With a bit of luck, the gains can become quite large. In theory the expected gain is even unlimited with

$$\text{total expected gain} = \frac{1}{2}\times 2^1 + \frac{1}{4}\times 2^2 + \frac{1}{8}\times 2^3 + \ldots = \sum_{n=1}^{\infty} \frac{1}{2^n}\times 2^n = \infty$$

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- Even in the worst case, the player will receive EUR 2. Bernoulli now asked "How much would you offer in order to be allowed to play this game with me?" The sums on offer were quite modest. This is the paradox (see `https://en.wikipedia.org/wiki/St._Petersburg_paradox#Expected_utility_theory`). The solutions on offer are two:
    1. Risk aversion or declining marginal utility of income: a certain loss, the price of entry, weighs more heavily than an uncertain gain, even if the latter is theoretically higher.
    2. Even a risk neutral person would only pay relatively modest amounts, if the bank's resources were limited (which in reality, of course, they are). That means gains are never infinite. Even if the casino's resources were EUR 1 billion, the value of the lottery would be only EUR 31!

- We are here interested in risk aversion, where the certainty of an average income, i.e., not participating in the bet, is preferred to the expected utility of doubling or losing that income.
- The graph below shows the corresponding utility curve.
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- A risk averse individual will always prefer to receive a stable income of EUR 30 to an arrangement where he received half the time EUR 10 and the other half of the time EUR 50.
- Expressed formally this yields

$$\underbrace{Utility(30)}_{\text{Case A}} > \underbrace{\frac{1}{2}Utility(10) + \frac{1}{2}Utility(5)}_{\text{Case B}}$$

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- Loss aversion is a third term covering the same phenomenon.
- The gain in the utility due to an additional unit of income is less than the loss of utility when losing the same amount.
- This form of preferences also explains insurance.
- Insurance guarantees a stable income.
- Imagine the case of a house with a value EUR 40 that can be insured against fire, that the probability of a fire during the lifetime of the house is $\frac{1}{2}$ and that the cost of insurance is hence EUR 20.
- If ensured, the owner will have a constant income of EUR 30 (Case A).
- If the owner does not take insurance, he will have with a probability of $\frac{1}{2}$ an income of EUR 50 and with a probability of $\frac{1}{2}$ an income of EUR 10.

If the owner is risk averse he will prefer to insure his house.

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- The graph provides further interesting information.
- Insurance firms aim at maximizing their profits.
- Insuring homes with a EUR 40 value that burn with a probability of $\frac{1}{2}$ for a price of EUR 20 will not allow making any profit given the unavoidable transaction costs.
- They will thus take a premium.
- This premium ΔI in the graph above corresponds to the difference between a stable income and the expected pay-off of risk taking, both yielding the same utility for the customer.
- In other words, if the insurance is capable of capturing for itself the complete benefits of insurance to the customer, its profits will be ΔI.
- As in the case of the franchise, it is probably wise to leave a portion of this surplus to the customer in order to internalise any negative externalities.

The impact of informational asymmetries

- George Akerlof showed in his famous paper “The Market for Lemons” how asymmetric information leading to suboptimal equilibria and a restriction or even disappearance of the market.
- Markets with asymmetric information concern used cars, insurance, restaurants etc. or any good whose value can only be observed after its acquisition.
- In “Equilibrium in Competitive Insurance Markets”, Rothschild and Stiglitz show how well-structured contracts can at least create submarkets to overcoming informational asymmetries.
- The market for car insurance is a good example.
- The insurer cannot distinguish between high risk- and low risk-drivers.
- Selling policies at average cost (average cost of accident times average probability) would cause losses, since the high-risk drivers buy more insurance than low-risk drivers.

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- If q indicates the quality of second-hand cars, where q is a random variable distributed uniformly on the interval [0, 1], then the average quality of a car is the expected value of q, i.e., 1/2.
- The market consists of a large number of buyers and sellers.
- The former have a reserve price (utility) equal to 3/2*q for vehicle of quality q, while sellers want to at least obtain q.
- In principle, there is thus ample space for gains of trade.
- It is assumed that buyers are risk neutral at the risk and ignorant of the quality of the vehicle in front of them. They know that q is uniformly distributed.
- Buyers thus calculates the expected quality with $q^e = \frac{1}{2}$.
- They are therefore willing to pay $3/2 \times 1/2 = \frac{3}{4}$ for the unknown car in front of them.

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- If the quality of each vehicle was observable, each seller would sell his vehicle at a price at least q, since the buyers are willing to pay 3/2*q, and each car would be sold at a price between q and 3/2*q.
- However, given that buyers offer $\frac{3}{4}$ regardless of the quality if the car sellers with vehicles with $q > \frac{3}{4}$ will leave the market.
- The quality of the remaining vehicles then lies between 0 and $\frac{3}{4}$.
- Buyers know this and adjust to $q^e = 3/8$. Henceforth, buyers will only be willing to pay $3/2*3/8 = 9/16$, causing the withdrawal of the sellers of cars whose quality is higher than $9/16$.
- By repeating this reasoning, it is easy to see that in the end remains only a small market for vehicles with $q = 0$, which are basically given away.

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- The paper by Rothschild and Stiglitz shows how insurers protect against informational asymmetry and market disappearance through a mechanism of self-selection in a separating equilibrium.
- The idea is to offer two contracts that are structured in a way that high risk drivers prefer the specific contract for high risks.
- This is achieved by reducing the amount of coverage of the contract offered to low risk (franchise).
- Insurers and high risk drivers maintain their utility in this arrangement, however, low risk drivers will only be able to obtain partial insurance.

The model

- There are two types of drivers with different probabilities of accident risk, pL (low risk) and pH (high risk).
- This is about adverse selection, not moral hazard.
- The initial wealth is W, and becomes either W_1 (without an accident) or W_2 (with an accident), damages are D, the probability of an accident is p.
- The insurance premium is α.
- The compensation received from the insurance in case of an accident is $\hat{a}$.

- It thus holds that:

	No accident	Accident
With insurance	$W_1 = W - \alpha$	$W_2 = W - D - \alpha + \hat{a}$
Without insurance	$W_1 = W$	$W_2 = W - D$

- The expected utility of an uninsured agent is thus:

$$\hat{U}_{SA} = (1 - p) * U(W) + p * U(W - D).$$

The expected utility of an insured agent instead is:

$$\hat{U}_{SA} = (1 - p) * U(W - \alpha) + p * U(W - D - \alpha + \hat{a}).$$

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- The model is driven by two concerns:
  1. The agents would like to smooth their utility in the two states W1 and W2 as they are risk averse.
  2. Insurers will need to make at least zero profits or $\Pi_I = \alpha - p * \hat{a} \geq 0$. With perfect competition it will hold that $\Pi = \alpha - p * \hat{a} = 0$ or $\alpha = p * \hat{a}$.

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- The slope of the line from point E (for "endowment" or initial wealth) is called the "fair odds-line".
- All along this line it holds under competition that $\alpha = p * \hat{a}$ or $\alpha = p * D$ with full insurance.
- Points below that line imply a gain for the insurer, points above that line it implies a loss.

- The slope of the fair odds-line (FOL) line is

$$Slope_{FOL} = -(D - \alpha/\alpha).$$

Under competition with $\alpha = p * D$ this becomes:

$$Slope_{FOL} = -(D - (p * D))/p * D = -(1 - p)/p.$$

- This implies the higher the probability of an accident, the more horizontal will be the fair odds-line.
- Due to risk aversion, agents seeking insurance will always seek out the tangency point between their indifference curves and the fair odds-line.

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- In the next case, the graph shows two different groups of agents seeking insurance, who have respectively accident probabilities that are either high (pH) or low (pL).
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- Under perfect information we obtain two equilibrium points for each of the groups.
- With asymmetric information, the problem is that high-risks will buy inexpensive insurance claiming that they are low risks.
- In the graph, it is visible that $\alpha_H > \alpha_L$ and that the indifference curve of the low risks is higher.
- Insurers however would go bankrupt as $\alpha_L < \hat{a} * pH$ assuming full insurance.

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- To protect themselves against asymmetric information and the disappearance of the market, insurers offer two contracts that will induce the two groups of customers to self-select.
- On offer will be the old contract for the high-risks, offering full protection but at a very high premium with $\alpha_H = pH * \hat{a}$, and a new contract for the low risks which only partially covers damages only partially with $\alpha_L = pL * \hat{a}_L$.
- This separating equilibrium (H/S) will force the high risks to choose their old contract as it would allow them a slightly higher utility (partial insurance is of no interest to them).
- Low risk customers will choose S as L is no longer on offer.

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- Compared to the full information equilibrium, low risk customers are considerably worse off, as they can only have partial insurance.
- Choosing H would mean vastly overpaying for insurance.
- The difference between equilibrium L and equilibrium S can be represented by the notion of a franchise (F) which corresponds to the distance between the net insurance desired by the low risks $(D - \alpha)$ and the amount offered by the insurance (DS).
- It therefore holds that $F = (D - \alpha) - D_S$.

Numerical example for calculating utilities in a separating equilibrium of two groups of agents with different accident risk

- The agents pL have an accident probability $1/3$;
- The agents pH have an accident probability $2/3$;
- The cost of insurance is α_H for pH and α_L for pL.
- Accident compensation is $\hat{a}_i$.
- Damage in case of an accident are D $= 24$.
- Both groups have an initial wealth of W $= 27$.

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- The utility function of both groups is W1*W2, where W1 is wealth in the absence of accident and W2 wealth in the case of an accident.
- This function indicates higher levels of utility for wealth levels that are similar in both states.
- There is decreasing utility of income for each state, which expresses risk aversion.

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- Their payoffs will be

	No accident	Accident
With insurance	$W_1 = 27 - \alpha_i$	$W_2 = 27 - 24 - \alpha_i + \hat{a}_i$
Without insurance	$W_1 = 27$	$W_2 = 27 - 24$

- The agents maximize their expected utilities according to the function:
- low probability agents

$$EUL = W1 * W2 = \underbrace{(1 - pL) * W1}_{\text{(without accident)}} + \underbrace{pL * W2}_{\text{(with accident)}}$$

- high probability agents

$$EUH = W1 * W2 = \underbrace{(1 - pH) * W1}_{\text{(without accident)}} + \underbrace{pH * W2}_{\text{(with accident)}} .$$

- A. Without insurance:

$$UL = (1 - pL) * W1 * pL * W2 = 2/3 * 27 * 1/3 * 3 = 18$$

$$UH = (1 - pH) * W1 * pH * W2 = 1/3 * 27 * 2/3 * 3 = 18$$

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- B. With insurance and symmetric information (full insurance)

$$UL = (1 - pL) * W1 * pL * W2 = 2/3 * (27 - \alpha_L) * 1/3 * (27 - 24 - \alpha_L + \hat{a}_L)$$

$$UH = (1 - pH) * W1 * pH * W2 = 1/3 * (27 - \alpha_H) * 2/3 * (27 - 24 - \alpha_H + \hat{a}_H$$

- To calculate âi (damage compensation in case of accidents, we know from the risk aversion of the agents that in equilibrium that they would like to buy the following amounts of insurance:

$$W1 = W2 \rightarrow W - \alpha_L = W - D - \alpha_L + \hat{a}_L \rightarrow \hat{a}_L = D = 24$$
$$\rightarrow W - \alpha_H = W - D - \alpha_H + \hat{a}_H \rightarrow \hat{a}_H = D = 24$$

- To calculate α_L and α_H, the insurance premiums for low and high risks, we know from the zero profit condition for insurers that it holds that

$$\Pi_L = \alpha_L - pL * D = 0 \rightarrow \alpha_L = pL * D = 1/3 * 24 = 8$$
$$\Pi_H = \alpha_H - pH * D = 0 \rightarrow \alpha_H = pH * D = 2/3 * 24 = 16.$$

- Substituting α_L, α_H, $\hat{a}_L$ and $\hat{a}_H$ in the utility function we obtain for the utility with full insurance

$$\begin{aligned} UL &= 2/3 * (27 - 8) * 1/3 * (27 - 24 - 8 + 24) \\ &= 19/3 * 38/3 = 722/9 \approx 80 > 18; \\ UH &= 1/3 * (27 - 16) * 2/3 * (27 - 24 - 16 + 24) \\ &= 11/3 * 22/3 = 242/9 \approx 26.9 > 18. \end{aligned}$$

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- In both cases it is profitable to conclude an insurance policy that guarantees the profitability of insurance ($\Pi \geq 0$).
- It also shows that low pH risk benefit considerably more from insurance than the high-risk.
- This is quite intuitive if one thinks about that risk aversion is about avoiding differences in income.
- The risk premium for the high risks is so high that one may almost forgo insurance.
- Having accidents is almost the normal case for high risks.
- Avoiding the damages from the rare event of an accident at reasonable costs is instead very attractive for the low risks.

- C. Under asymmetric information
- The problem is that customers with pH prefer to buy the insurance of customers with pL, which would give them with great utility:

$$UH(L) = 2/3 * (27 - 8) * 1/3 * (27 - 24 - 8 + 24)$$
$$= 38/3 * 19/3 = 722/9 >> 242/9.$$

- This would however be unprofitable for insurers:

$$\Pi_H(L) = \alpha_L - pH * D = 8 - 2/3 * 24 = 8 - 16 = -8 < 0.$$

- Since insurers do not have any way of distinguishing between pL and pH insurers would therefore withdraw from the market and no insurance at all would be available.

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- D. The solution: Separating Equilibrium with self-selection
- Insurers will offer a contract to pL with a lower level of utility for the pH than their initial contract ($UH = 242/9 \approx 27$).
- Thus the pH will choose their initially offered contract UH originally proposed.
- The contract U_L^S (i.e., the new contract offered to low risk, now less advantageous in terms of risk coverage, allowing to establish a separating equilibrium) must still always satisfy the requirement that insurers make no losses.
- It there for holds that:

$$\alpha_L = p_L * \hat{a}_L = 1/3\hat{a}_L$$

and thus

$$\hat{a}_L = 3 * \alpha_L$$

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- To determine α_L, the price or premium of the new contract it thus holds that

$$\begin{aligned} ULS \leq UH = 27 &= 2/3 * (27 - \alpha_L) * 1/3 * (27 - 24 - \alpha_L + 3 * \alpha_L) \\ &= (18 - 2/3 * \alpha_L) * (1 + 2/3 * \alpha_L) \\ &= 18 - 2/3 * \alpha_L + 12\alpha_L - 4/9 * \alpha_L^2 \\ &= 18 + 34/3 * \alpha_L - 4/9 * \alpha_L^2 = 27 \end{aligned}$$

- and thus

$$\alpha_L^2 - 52/2 * \alpha_L + 81/4 = 0.$$

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- This gives the solution for the premium α_L of the contract offered to the low risks:
  - $\alpha_{L1} \approx (25.5 - 23.85)/2 = 0.82$: correct amount
  - $\alpha_{L2} \approx (25.5 + 23.85)/2 = 24.67$: this would yield a damage compensation claim of $\hat{a}_L = 3 * \alpha_{L2} = 3 * 24.67 = 74.01 >> 24$, which is far higher than the real damages.
- Insurers will thus only offer a slightly lower amount at, say, $\hat{a}_L = 3 * 0.8 = 2.40 << D = 24$, which is far lower than the real damages.
- The difference is particularly large due to the shape of the utility function.
- In the case of an accident, there is now a franchise (i.e., an uninsured amount) of 21.60 to be paid.
- The low risks pL therefore obtain in this separating equilibrium only very partial coverage.

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- In this new contract the pL and the pH would obtain the following utility:

$$UL = UH = 2/3 * (27 - 0.8) * 1/3 * (27 - 24 - 0.8 + 2.40)$$
$$= 2/3 * 26.2 * 1/3 * 4.6 = 26.8 < 26.9.$$

- The pH and pL would get the same utility under the new contract.
- However, the pH can do better with full coverage at $\alpha_L = 16$ and $\hat{a}_L = D = 24$ and $UH = 27$.
- They will thus stay on their original contract corresponding to the symmetric information situation.
- In choosing it, they will reveal themselves as pH.

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- The pL lose a considerable amount of utility, as they obtained in the full insurance case an utility of 80.
- Nevertheless, their utility at 26.9 remains higher than the one they would have obtained in the case without insurance at $UL = 18$, since they have obtained at least partial insurance.