Online Supplement for "Percentage Fees in Thin Markets: An Empirical Perspective" by Simon Loertscher and Andras Niedermayer

H Common Value Component

In this appendix, we extend our model to account for common values between sellers and buyers. Our analysis rests on the linear, additively separable specification of Cai, Riley, and Ye (2007, Section V) and Jullien and Mariotti (2006).

H.1 Setup

One-Shot Model As in the independent private values model, the seller's type c is the seller's opportunity cost of selling (in the static model, that is, with $\delta = 0$) or his cost of production, with the distribution of c being $G(c)$ with support $[\underline{c}, \overline{c}]$ and density $g(c) > 0$ for all $c \in (\underline{c}, \overline{c})$. A buyer's type is now denoted x, where x is assumed to be distributed according to $F(x)$ with support $[\underline{v}, \overline{v}]$ and density $f(x) > 0$ for all $x \in (\underline{v}, \overline{v})$. All types are assumed to be independently distributed. The willingness to pay $v(x, c)$ of a buyer of type x who buys from a seller of type c is given by

$$v(x, c) := \lambda c + (1 - \lambda)x, \tag{15}$$

where $\lambda \in [0, 1]$ measures the severity of the common value component, with $\lambda = 0$ corresponding to the private values model and $\lambda = 1$ representing the pure common value model.

Dynamic Model with Percentage Fees The model we analyze is a dynamic version of the linear model sketched above and analyzed by Cai, Riley, and Ye (2007), which we augment by percentage fees.[55] The dynamic environment is the same as in the main body of the paper: In every period a random number of buyers arrive, drawing their types x_b independently from F with support $[\underline{x}, \overline{x}]$, and we assume that the realized number of

[55] A similar static model without percentage is analyzed by Jullien and Mariotti (2006) for the case of two buyers.

buyers is not known by the seller when he sets the reserve.[56] We impose stationarity and assume that the common discount factor is $\delta \in [0,1)$. Moreover, buyers do not condition their behavior on past reserve prices set by the seller. A simple assumption that implies this restriction is that they do not observe these prices.[57] As in the main text, we assume that the function $\Phi(x) := x - \frac{1-F(x)}{f(x)}$ is increasing.

Equilibrium Next we derive the separating equilibrium in the dynamic model with percentage fees $\omega(p) = bp$ with $b \in [0,1)$. Let x be the buyer type who is indifferent between buying and not buying when the price is $\lambda\hat{c} + (1-\lambda)x$ and the seller is believed to be of type $\hat{c}$. Denote by $W_S^{cv}(c, \hat{c}, x)$ the discounted expected payoff of a seller in the common value setup when the seller's type is c, buyers believe that his type is $\hat{c}$, and the indifferent buyer type is x. Given the belief that the seller is of type $\hat{c}$, the willingness to pay of a buyer of type x is $\lambda\hat{c} + (1-\lambda)x$. Given x and $\hat{c}$, this will thus be the reserve price. Consequently, we have

$$\begin{aligned} W_S^{cv}(c, \hat{c}, x) &= \delta W_S^{cv}(c, \hat{c}, x) F_{(1)}(x) + [(1-b)(\lambda\hat{c} + (1-\lambda)x) - c][F_{(2)}(x) - F_{(1)}(x)] \\ &+ \int_x^{\overline{x}} [(1-b)(\lambda\hat{c} + (1-\lambda)y) - c] dF_{(2)}(y), \end{aligned}$$

where b is the proportional fee, that is $w(p) = bp$ with $b < 1$. The first summand is the expected discounted continuation value if no sale occurs today. The other two summands capture the expected payoff from a sale in the current period, in which case the game ends. Solving for $W_S^{cv}(c, \hat{c}, x)$ we get

$$\begin{aligned} W_S^{cv}(c, \hat{c}, x) &= \frac{1}{1 - \delta F_{(1)}(x)} \Big\{ [(1-b)(\lambda\hat{c} + (1-\lambda)x) - c][F_{(2)}(x) - F_{(1)}(x)] \\ &+ \int_x^{\overline{v}} [(1-b)(\lambda\hat{c} + (1-\lambda)y) - c] dF_{(2)}(y) \Big\}, \end{aligned}$$

[56] In contrast, Cai, Riley, and Ye (2007), and Jullien and Mariotti (2006), assume that the number of buyers is deterministic and commonly known. Similarly to Jullien and Mariotti (2006) we assume that apart from the seller's information about quality, we are in a private value environment, so that an English auction and a second-price auction are equivalent.

[57] But even if they observed them, there would still be a perfect Bayesian equilibrium in which these prices are completely ignored on the equilibrium path.

which can be written more compactly, using the notation $1 - F_\infty(y) = \frac{1-F_{(1)}(y)}{1-\delta F_{(1)}(y)}$ and $R(x) = \frac{x[F_{(2)}(x)-F_{(1)}(x)]+\int_x^{\overline{v}} y dF_{(2)}(y)}{1-F_{(1)}(x)}$ introduced in the main text, as

$$W_S^{cv}(c, \hat{c}, x) = (1-b)(1-F_\infty(x)) \left[(1-\lambda)R(x) + \lambda\hat{c} - \frac{c}{1-b}\right]. \tag{16}$$

Let $W_{Si}^{cv}(.,.,.)$ denote the first derivative of W_S^{cv} with respect to its ith argument and $W_{Sij}^{cv}(.,.,.)$ the cross partial, that is, the derivative of $W_{Si}^{cv}(.,.,.)$ with respect to its jth argument. Observe that

$$W_{S2}^{cv}(c, \hat{c}, x) = (1-b)\lambda(1-F_\infty(x)) > 0 \tag{17}$$

and

$$W_{S3}^{cv}(c, \hat{c}, x) = -(1-b)f_\infty(x) \left[(1-\lambda)\tilde{\Phi}(x) - \lambda\hat{c} - \frac{c}{1-b}\right], \tag{18}$$

where $\tilde{\Phi}(x) := R(x) - \frac{1-F_\infty(x)}{f_\infty(x)}R'(x)$ as in the main text.

Notice that in equilibrium x will depend on $\hat{c}$, so that we can write $x = x(\hat{c})$.[58] A necessary condition for a separating equilibrium is

$$W_{S2}^{cv}(c, c, x(c)) + W_{S3}^{cv}(c, c, x(c))x'(c) = 0. \tag{19}$$

Equivalently, (19) can be expressed as

$$W_{S2}^{cv}(c(x), c(x), x)c'(x) + W_{S3}^{cv}(c(x), c(x), x) = 0, \tag{20}$$

where $c(x)$ is the inverse of $x(c)$. The initial condition is

$$W_{S3}^{cv}(\underline{c}, \underline{c}, x^*(\underline{c})) = 0.$$

The results in Cai, Riley, and Ye (2007) imply that a separating equilibrium exists and is unique if $W_{S31}^{cv} > 0$ and if the function

$$\beta(c, x) := -\frac{W_{S3}^{cv}(c, c, x)}{W_{S2}^{cv}(c, c, x)} = \frac{f_\infty(x)}{\lambda(1-F_\infty(x))} \left[(1-\lambda)\tilde{\Phi}(x) - \frac{1-\lambda(1-b)}{1-b}c\right] \tag{21}$$

[58]For the linear specification we have, $x(\hat{c}) = \hat{c}$.

intersects with 0 at most once and, if it does, from below as x increases.[59] Observe that the equality in (21) follows by substituting the expressions for $W^{cv}_{S2}(c, \hat{c}, x)$ and $W^{cv}_{S3}(c, \hat{c}, x)$ evaluated at $\hat{c} = c$.

We are now going to show that the conditions identified by Cai, Riley, and Ye (2007) are satisfied in our setup. To that end, notice first that

$$W^{cv}_{S31}(c, c, x) = f_\infty(x) > 0.$$

Second, the function $\beta(c, x)$ will change its sign at most once (from negative to positive) as x increases if $\tilde{\Phi}(x)$ is increasing. We are now going to show that monotonicity of $\Phi(x)$ implies monotonicity of $\tilde{\Phi}(x)$. To see this, observe first that $R'(x) = \frac{f_{(1)}(c)}{1-F_{(1)}(c)}[R(x) - \Phi(x)] > 0$ and that therefore

$$\tilde{\Phi}(x) = R(x)\left[1 - \frac{1 - \delta F_{(1)}(x)}{1-\delta}\right] + \frac{1 - \delta F_{(1)}(x)}{1-\delta}\Phi(x).$$

Consequently,

$$\tilde{\Phi}'(x) = R'(x)\left[1 - \frac{1 - \delta F_{(1)}(x)}{1-\delta}\right] + \frac{\delta}{1-\delta} f_{(1)}(x)[R(x) - \Phi(x)] + \frac{1 - \delta F_{(1)}(x)}{1-\delta}\Phi'(x) > 0,$$

which is positive because each summand is positive. We have thus verified that the conditions Cai, Riley, and Ye (2007) hold in our setup.

Proposition 5. *(i) There is a unique separating equilibrium in which buyers only condition on the current-period reserve price.*

(ii) In this equilibrium, the marginal type $x^*(c)$ is given by the differential equation

$$-\frac{1 - F_\infty(x^*(c))}{f_\infty(x^*(c))}\lambda + x^{*\prime}(c)\left\{(1-\lambda)\tilde{\Phi}(x^*(c)) - \left(\frac{1}{1-b} - \lambda\right)c\right\} = 0 \qquad (22)$$

with the initial condition given by

$$\tilde{\Phi}(x^*(\underline{c})) = \frac{1 - (1-b)\lambda}{(1-b)(1-\lambda)}\underline{c}. \qquad (23)$$

[59] As mentioned, Cai, Riley, and Ye (2007) do not analyze a model with a stochastic number of buyers as we do. However, a careful reading of their paper reveals that only the distributions $F_{(1)}(x)$ and $F_{(2)}(x)$ of the highest and the second-highest buyer type matter for their analysis. Therefore, this analysis extends directly to the setup with a stochastic number of buyers. Moreover, because there are no informational externalities between buyers in our setup, the additional conditions identified by Lamy (2010) are not needed.

(iii) Equivalently, the inverse marginal type function $c^*(x)$ in this equilibrium is given by the differential equation

$$-c'(x)\frac{1-F_\infty(x^*(c))}{f_\infty(x^*(c))}\lambda + x^{*\prime}(c)\left\{(1-\lambda)\tilde{\Phi}(x^*(c)) - \left(\frac{1}{1-b}-\lambda\right)c(x)\right\} = 0$$

with the initial condition

$$\tilde{\Phi}(\underline{x}^*) = \frac{1-(1-b)\lambda}{(1-b)(1-\lambda)}\underline{c}.$$

Proof of Proposition 5. Part (i) follows from Theorem 1 in Cai, Riley, and Ye (2007), and part (iii) follows from part (ii). We are therefore left to prove part (ii).

Substituting the expressions for the derivatives from (17) and (18) into the first-order condition (19), dividing by $f_\infty(x^*(c))$ and making the substitution $\tilde{\Phi}(x) = R(x) - \frac{1-F_\infty(x)}{f_\infty(x)}R'(x)$ yields (22). Making the same substitutions, the initial condition $W^{cv}_{S3}(\underline{c}, \underline{c}, x^*(\underline{c})) = 0$ becomes (23). The arguments by Cai, Riley, and Ye (2007) can be applied with minimal adjustments to account for the percentage fee b and to show that, under the given assumptions, the solution to the initial condition and the differential equation is unique and characterizes a profit maximum. □

An important difference between the private values model and the separating equilibrium of the model with a common value component is that conditional on the buyer's type x, the transaction price $\breve{p}$ will vary with the reserve price p if there is a common value component. We now derive the distribution of the transaction price $\breve{p}$ conditional on the reserve price p, which we denote $\breve{F}(\breve{p}|p)$.

Let

$$P(c) := \lambda c + (1-\lambda)x^*(c)$$

be the strictly increasing reserve price function in the separating equilibrium. Let $x(\breve{p}, p)$ be the buyer type who is indifferent between buying and not buying when the transaction price is $\breve{p}$ and the reserve price is p. This indifferent type is given by

$$x(\breve{p}, p) = \frac{\breve{p} - \lambda P^{-1}(p)}{1-\lambda}.$$

Let

$$x(p) := x(p, p)$$

be the buyer type who is indifferent between buying and not buying at the reserve p.

Conditional on a transaction occurring, the probability that the transaction price is equal to the reserve price is thus

$$\Pr(\breve{p} = p) = \frac{F_{(2)}(x(p)) - F_{(1)}(x(p))}{1 - F_{(1)}(x(p))}.$$

The probability that the transaction price $\breve{p}$ satisfies $\breve{p} \leq \breve{p}_0$ for some $\breve{p}_0 > p$ is

$$\Pr(\breve{p} \leq \breve{p}_0) = \frac{F_{(2)}(x(\breve{p}_0, p)) - F_{(2)}(x(p))}{1 - F_{(1)}(x(p))}.$$

The cumulative distributions of transaction prices $\breve{p}$ conditional on trade occurring given reserve p is thus given by

$$\breve{F}(\breve{p}|p) = \begin{cases} 0 & \text{if } \breve{p} < p \\ \frac{F_{(2)}(x(p)) - F_{(1)}(x(p))}{1 - F_{(1)}(x(p))} & \text{if } \breve{p} = p \\ \frac{F_{(2)}(x(\breve{p}, p)) - F_{(1)}(x(p))}{1 - F_{(1)}(x(p))} & \text{if } \breve{p} \in (p, \overline{x}], \\ 1 & \text{if } \breve{p} > \overline{x}. \end{cases}.$$

H.2 Identification

We now show non-parametric identification in a model with a common value component, restricting our attention to identification without unobserved heterogeneity. Observe that with independent private values (see Section 6) we have shown non-parametric identification without making use of the transaction prices. The basic idea of the following identification argument is that we use additional information on transaction prices to identify the weight of the common value component λ. Informally, under independent private values ($\lambda = 0$), a higher reserve does not change the distribution of transaction prices, except for a truncation. In the presence of a common value component, the distribution changes beyond a truncation as the reserve price increases. We state this idea more formally in the following.

Let $1 - H_\infty(p)$ be the discounted probability of sale given reserve price p, which is empirically observable.[60] Further, denote by $1 - \overline{H}(k)$ the discounted probability of sale with a common value, given the average transaction price k. The maximization problem

[60] H is non-parametrically identifiable through the relation between the reserve and whether a property was sold.

for a seller of type c is then equivalent to choosing k to maximize $((1-b)k-c)(1-\overline{H}(k))$. The first-order condition is $(1-b)\left(k-\frac{1-\overline{H}(k)}{\overline{h}(k)}\right)-c=0$, yielding

$$\overline{P}^{-1}(k) := (1-b)\left(k-\frac{1-\overline{H}(k)}{\overline{h}(k)}\right)$$

as the inverse of the optimal average transaction price rule.

Let $R(p)$ be the average transaction price in any period conditional on a transaction occurring in this period, given the reserve p. Then $k = R(p)$ and, under monotonicity, $p = R^{-1}(k)$. Note that $R(p)$ is identifiable from the joint distribution of the reserve prices and the transaction prices.

The probability $q_p(\breve{p})$ that the transaction price is larger than $\breve{p}$ for some $\breve{p} > p$ is then

$$q_p(\breve{p}) = 1 - F_{(2)}\left(x(\breve{p}, p)\right).$$

This probability $q_p(\breve{p})$ is observable. Inverting $q_p(\breve{p}) = 1 - F_{(2)}\left(x(\breve{p}, p)\right)$, we get

$$q_p^{-1}(q) = \lambda P^{-1}(p) + (1-\lambda)F_{(2)}^{-1}(1-q).$$

Evaluating $q_p^{-1}(q)$ at two different reserve prices p_1 and p_2 and taking differences yields

$$q_{p_2}^{-1}(q) - q_{p_1}^{-1}(q) = \lambda[P^{-1}(p_2) - P^{-1}(p_1)].$$

Therefore, λ is identified by

$$\lambda = \frac{q_{p_2}^{-1}(q) - q_{p_1}^{-1}(q)}{P^{-1}(p_2) - P^{-1}(p_1)}. \tag{24}$$

Using

$$\begin{aligned}
\overline{H}(k) &= H_\infty(R^{-1}(k)) \\
\overline{P}^{-1}(k) &= \left(k-\frac{1-\overline{H}(k)}{\overline{h}(k)}\right)(1-b) \\
P^{-1}(p) &= \overline{P}^{-1}(R(p)).
\end{aligned}$$

and plugging this into (24) gives us λ. With λ at hand, one can use the same approach as for independent private values (see Section 6) to show identification of F, G, δ, and τ.

H.3 Likelihood Function

We now derive the likelihood function.

No Unobserved Heterogeneity Assume first that there is no unobserved heterogeneity. Let

$$g_p(p) := g(P^{-1}(p))[P^{-1}(p)]'$$

denote the density of reserve prices.

The probability mass function given reserve p for time on market t with $t = 0, .., \infty$ is then given, if the property sell, by

$$[\delta F_{(1)}(x(p))]^{t/\tau}(1 - F_{(1)}(x(p)))$$

and if the property does not sell, by

$$[\delta F_{(1)}(x(p))]^{t/\tau}(1 - \delta)F_{(1)}(x(p)).$$

Let $S = 1$ if the object ever sells and $S = 0$ if it never sells. Let $h_{tpS}(t, p, S)$ denote the likelihood of observing (t, p, S). This function is given by

$$h_{tpS}(t, p, S) = \begin{cases} [\delta F_{(1)}(x(p))]^{t/\tau}(1 - F_{(1)}(x(p)))g_p(p) & \text{if } S = 1 \\ [\delta F_{(1)}(x(p))]^{t/\tau}(1 - \delta)F_{(1)}(x(p))g_p(p) & \text{if } S = 0 \end{cases}.$$

Next, consider transaction prices $\breve{p}$. Let $h_{\breve{p}}(\breve{p}|p, S)$ denote the likelihood of observing $\breve{p}$ given (p, S) and set $\breve{p} = 0$ if $S = 0$. Under the assumption of stationarity, $h_{\breve{p}}(\breve{p}|p, S)$ is independent of t. By the previous arguments,

$$h_{\breve{p}}(\breve{p}|p, S) = \begin{cases} \breve{f}(\breve{p}|p) & \text{if } S = 1 \\ \Delta(\breve{p}) & \text{if } S = 0 \end{cases},$$

where $\Delta(.)$ is the Dirac delta function and where the density $\breve{f}(\breve{p}|p)$ can be written as

$$\breve{f}(\breve{p}|p) = \begin{cases} 0 & \text{if } \breve{p} < p \\ \frac{F_{(2)}(x(p)) - F_{(1)}(x(p))}{1 - F_{(1)}(x(p))}\Delta(\breve{p} - p) & \text{if } \breve{p} = p, \\ \frac{f_{(2)}(x(\breve{p}, p))\frac{1}{1-\lambda}}{1 - F_{(1)}(x(p))} & \text{if } \breve{p} > p \end{cases}$$

Let $\boldsymbol{X}_i = (t_i, p_i, S_i, \breve{p}_i)$ be an observation. Then the likelihood function $l(\boldsymbol{X}_i|\boldsymbol{\theta})$ in the absence of unobserved heterogeneity is

$$l(\boldsymbol{X}_i|\boldsymbol{\theta}) = h_{tpS}(t, p, S)h_{\breve{p}}(\breve{p}|p, S),$$

where $\boldsymbol{\theta}$ is the vector of parameters determining the shape of h_{tpS} and $h_{\breve{p}S}$. Observe that the private values model is nested as the special case of the above with $\lambda = 0$ (which implies $x(\breve{p}, p) = \breve{p}$).

Unobserved Heterogeneity To account for unobserved heterogeneity, we now extend the setup by distinguishing between the (true) transaction price $\breve{p}_i$ and the (true) reserve price p_i on the one hand and, as in the private values model, between the observed transaction price of object i, which we denote $\hat{\breve{P}}_i$, and the *observed listing price*, which we denote $\hat{P}_i$, on the other hand. Like in the private values model, we stipulate that there is a multiplicative quality index ϑ_i for every object i, which is observed by all market participants but not directly observable by the econometrician. The econometrician only observes a proxy $\hat{\vartheta}_i$, given by

$$\hat{\vartheta}_i = \frac{P_q^{\text{index}}}{P_{q'}^{\text{index}}} \hat{P}_{iq'},$$

where q' and q denote two different quarters and P_q^{index} is the price index in quarter q. The relationship between ϑ_i and $\hat{\vartheta}_i$ is given by

$$\hat{\vartheta}_i = \epsilon_i^Q \vartheta_i,$$

where ϵ_i^Q is the unobserved heterogeneity in quality.

In addition, in the model with a common value component there is the information c about the true objective quality of the object that a priori is only known by the seller but is assumed to be correctly inferred by any bidder in the separating equilibrium of the English auction with reserve price signaling. From the perspective of bidders, it seems natural to refer to ϑ as the *observed quality* and to c as the *inferred quality* of an object (as mentioned, we observe neither directly). The willingness to pay for an object of observed quality ϑ and inferred quality c for a buyer of type x is

$$w(x, c, \vartheta) := v(x, c)\vartheta,$$

where $v(x, c) = \lambda c + (1 - \lambda)x$ is as defined in (15).

If the true transaction price $\breve{p}_i$ for object i is determined by an indifferent buyer of type x, as is the case in an English auction when the transaction occurs at a price above the reserve, we have

$$\breve{p}_i = v(x, c_i),$$

where $c_i = P_\lambda^{-1}(p_i)$ is the inferred type of the seller of object i who sets the true reserve p_i.

Let $\breve{P}_i$ be the *observed quality-adjusted transaction price*. This is the price observed by the econometrician. It satisfies

$$\breve{P}_i = \frac{\hat{\breve{P}}_i}{\hat{\vartheta}_i} = \breve{p}_i \epsilon_i^Q \quad \text{or equivalently} \quad \breve{p}_i = \frac{\breve{P}_i}{\epsilon_i^Q}.$$

Let p_i be the "true" *reserve price* for object i, with "true" meaning that the reserve satisfies $p_i \vartheta_i$. We assume that p_i is observed without errors by the market participants, but observed with a "discount" noise ϵ_i^D by the econometrician, satisfying the relationship

$$\hat{P}_i = \vartheta_i p_i \epsilon_i^D,$$

where $\hat{P}_i$ is the *listing price* the econometrician observes. The listing price P_i constructed by the econometrician, who also observes the quality proxy $\hat{\vartheta}_i$, is therefore such that

$$P_i = \frac{\hat{P}_i}{\hat{\vartheta}_i} = p_i \epsilon_i^D \epsilon_i^Q.$$

Letting $\epsilon_i^P := \epsilon_i^D \epsilon_i^Q$, we thus have

$$p_i = \frac{P_i}{\epsilon_i^P},$$

and a typical observation is given by

$$\boldsymbol{X}_i = (T_i, P_i, S_i, \breve{P}_i).$$

The likelihood function that accounts for unobserved heterogeneity is then given by

$$l(\boldsymbol{X}_i|\boldsymbol{\theta}) = \sum_{k=1}^{T_i/\tau} \int_0^\infty \int_0^\infty h_{tpS}(T_i - k\tau, P_i/\epsilon^P, S_i) h_{\breve{p}}(\breve{P}_i/\epsilon^Q | P_i/\epsilon^P, S_i) h_t(k\tau) h_Q(\epsilon^Q) h_D(\epsilon^D) d\epsilon^Q d\epsilon^D, \tag{25}$$

where $\breve{P}_i$ is the constructed transaction price; $h_j(\epsilon^j)$ is the density of the error term ϵ^j with $j \in \{Q, D\}$; k is the summation variable; and τ represents the error term ϵ^T. Rewriting (25) in terms of ϵ^D and ϵ^P using $\epsilon^Q = \epsilon^P/\epsilon^D$ we get

$$l(\boldsymbol{X}_i|\boldsymbol{\theta}) = \sum_{k=1}^{T_i/\tau} \int_0^\infty \int_0^\infty h_{tpS}(T_i - k\tau, P_i/\epsilon^P, S_i) h_{\breve{p}}(\breve{P}_i/(\epsilon^P/\epsilon^D) | P_i/\epsilon^P, S_i) h_t(k\tau) h_P(\epsilon^P) h_D(\epsilon^D) d\epsilon^D d\epsilon^P, \tag{26}$$

where $h_P(\epsilon^P)$ is the density of ϵ^P.

H.4 Estimation

The following describes the general estimation procedure for the model with a common value component. With the exception of one simplification that we impose for the purpose of speeding up computations and that is spelled out at the end of this subsection, this is how we estimated the model when allowing for a positive common value component λ.

Define

$$y_B := \frac{2x - \underline{x} - \overline{x}}{\overline{x} - \underline{x}} \quad \text{and} \quad y_S := \frac{2c - \underline{c} - \overline{c}}{\overline{c} - \underline{c}}$$

as the valuation and cost normalized to the range $[-1, 1]$ used for Chebyshev polynomials. Virtual valuation and virtual cost functions are parameterized with the Chebyshev polynomials

$$\Phi(x) = \sum_{i=0}^{N} \phi_i \mathcal{T}_i(y_B) \quad \text{and} \quad \Gamma(c) = \sum_{i=0}^{N} \gamma_i \mathcal{T}_i(y_S), \tag{27}$$

where $\mathcal{T}_i(x)$ is the degree i Chebyshev polynomial and N the degree of polynomial approximation.[61]

We impose the restriction $\underline{x} = \underline{c}$ and $\overline{x} = \overline{c}$ and only use the parameters $\underline{c}$ and $\overline{x}$ in the following. (Note that for the independent private values model in the main text we simply use v instead of x and have $\underline{v} = \underline{x}$, $\overline{v} = \overline{x}$.) δ is the discount factor. The error in the time on market ϵ^T follows a geometric distribution with survival rate β_T. The reserve price noise ϵ^P and the discount noise ϵ^D are lognormally distributed with $\ln \epsilon^P \sim N(\mu_D, \sigma_P)$ and $\ln \epsilon^D \sim N(\mu_D, \sigma_D)$. (The underlying assumption is that the log of unobserved heterogeneity $\ln \epsilon^Q \sim N(0, \sigma_Q)$ and that $\ln \epsilon^P = \ln \epsilon^Q + \ln \epsilon^D$. We will use (ϵ^P, ϵ^D) rather than (ϵ^Q, ϵ^D) in our estimation. We have the restriction $\sigma_P > \sigma_D$.)

The parameters of the empirical model are thus $\{\phi_i\}_{i=0}^N$, $\{\gamma_i\}_{i=0}^N$, β_T, σ_p, δ, $\underline{v}$, $\overline{v}$, $\underline{c}$, $\overline{c}$, ξ, μ_D, σ_D, and λ.

[61] The sequence of Chebyshev polynomials starts with $\{\mathcal{T}_i(y)\}_{i=0} = \{1, y, 2y^2 - 1, 4y^3 - 3y, 4y^4 - 8y^2 + 1, ...\}$

F and G are obtained from their closed-form solutions given the parameterization of Φ and Γ as Chebyshev polynomials. The distributions $F_{(1)}$, $F_{(2)}$ and F_∞, their corresponding densities, and $\tilde{\Phi}$ are then given in exactly the same ways as in the private values model.

To satisfy the constraints $\Phi(\overline{x}) = \overline{x}$ and $\Gamma(\underline{c}) = \underline{c}$, we set the parameters ϕ_0 and γ_0 such that

$$\phi_0 = \overline{x} - \sum_{i=1}^{N} \phi_i \quad \text{and} \quad \gamma_0 = \underline{c} - \sum_{i=1}^{N} \gamma_i (-1)^i. \tag{28}$$

To see that this is correct, observe that $x = \overline{x}$ corresponds to $y_B = 1$ and $c = \underline{c}$ corresponds to $y_S = -1$, and that $\mathcal{T}_i(1) = 1$ and $\mathcal{T}_i(-1) = (-1)^i$ for all i.

The likelihood function $l(\boldsymbol{X}|\boldsymbol{\theta})$ used for the estimation, given observations $\boldsymbol{X} = (\boldsymbol{X}_i)_{i=1}^n$, is then given by

$$l(\boldsymbol{X}|\boldsymbol{\theta}) = \prod_{i=1}^{n} l(\boldsymbol{X}_i|\boldsymbol{\theta}),$$

where the $l(\boldsymbol{X}_i|\boldsymbol{\theta})$ are given by (26) and where

$$\boldsymbol{\theta} = (\{\phi_i\}_{i=0}^N, \{\gamma_i\}_{i=0}^N, \beta_T, \sigma_p, \delta, \underline{v}, \overline{v}, \underline{c}, \overline{c}, \xi, \mu_D, \sigma_D, \lambda)$$

is the vector of parameters over which we compute Bayesian posterior expectations of $l(\boldsymbol{X}|\boldsymbol{\theta})$. As in the main text, we assume an uninformative Bayesian prior, so that the Bayesian posterior belief is proportional to the likelihood function.

To avoid the double complexity of estimating a common value component model and non-linear virtual type functions (i.e. non-generalized Pareto distributions), we restrict attention to linear virtual type functions. This is sufficient for the purpose of this section in the appendix, that is, of estimating the weight of the common value component λ. Of course, it would not be sufficient for the purpose of estimating how good an approximation of G a mirrored generalized Pareto distribution is.

The results are displayed in Table 12. For each of the three years, we find $\lambda \approx 0.06$. This means that if the seller's cost increases by \$1,000, the gains from trade decrease by \$940 and every buyer's willingness to pay increases by \$60. In other words, the idiosyncratic component that reduces gains from trade seems to clearly dominate the common value component. Although λ is not exactly 0, it seems small enough to be abstracted from our analysis in the main text. There are two possible and plausible forces at play that make the common value component small in the market we study. First, our data set contains only condominium properties. This submarket is more homogenous than the real estate market as a whole, which plausibly renders common value concerns less important. Second, this is a market with voluntary participation, which can only function well if the adverse selection problem is not too severe.[62]

Estimated Parameter Values						
Parameters	1993		1994		1995	
ϕ_1	0.981	(0.0434)	1.04	(0.0570)	0.962	(0.0331)
γ_1	1.25	(0.0344)	1.24	(0.0306)	1.17	(0.0240)
$\underline{x}$	0.0149	(0.0143)	0.0144	(0.0141)	0.0103	(0.0101)
$\overline{x}$	1.29	(0.0255)	1.28	(0.0224)	1.20	(0.0148)
δ	0.947	(0.0127)	0.948	(0.0191)	0.984	(0.00114)
ξ	0.0645	(0.0841)	0.158	(0.169)	0.0647	(0.0581)
λ	0.0631	(0.0223)	0.0619	(0.0183)	0.0624	(0.0142)
μ_D	0.0980	(0.00505)	0.0843	(0.00340)	0.0735	(0.00235)
σ_D	0.103	(0.00353)	0.0740	(0.00237)	0.0572	(0.00167)
σ_P	0.260	(0.00792)	0.233	(0.00659)	0.230	(0.00638)
β_T	0.0718	(0.0196)	0.101	(0.0480)	0.380	(0.0563)
# Observations	727		830		787	

Table 12: Estimated parameter values for 1993 to 1995 with a common value component. Table entries read: Mean (standard deviation).

[62] This also provides an explanation for the different results obtained by Niedermayer, Shneyerov, and Xu (2015), who study foreclosure real estate auctions, which do not exhibit voluntary participation.