A Search Model of Rental Markets: Who Should Pay the Commission?*

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Abstract

We develop a dynamic search equilibrium model of rental markets with brokerage fees. An endogenous mass of landlords and tenants is active in the search market. While the mass of entering tenants is exogenous, the mass of entering landlords is endogenously determined by the rate at which apartments become vacant. Brokerage fees paid by tenants are used as a device to screen short term and long term tenants. We analyze a policy that requires fees to be paid by landlords. Such a policy decreases efficiency by preventing efficient screening, but also increases turnover and hence efficiency in the search market. We further provide empirically testable implications of our theory.

Keywords: Search, dynamic random matching, brokerage fees, appointment principle

JEL-Classification: D82, D83, R31

1 Introduction

Rental markets are important, since rents comprise a significant fraction of consumers' expenditures (rent expenses account for roughly one third of total expenses in the U.S. and roughly one quarter of expenses in Germany). Recently, the question who should pay real estate brokers' fees in rental market has received considerable attention by the public and by policy makers. In Germany, in June 29, 2015, after considerable public

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controversy, a law came into effect which bans fees being charged to tenants and requires landlords to pay the fees – the "Appointment Principal" ("Bestellerprinzip").[1] While no such laws have been made in the U.S., there has been academic interest in who pays the fee in rental markets, with the landlords paying the fee in some submarkets and tenants paying the fee in others.[2]

We develop a search equilibrium model to analyze the effect of which party pays the brokerage fees. In an infinite horizon model, there is a fixed mass of landlords who own apartments. Landlords whose tenants move out of their apartments enter the search market. In every period there is an inflow of potential tenants who search for an apartment. A crucial feature of the model is that tenants are heterogeneous with respect to whether they are short term tenants who move out after a short period or long term tenants. Landlords are also heterogeneous with respect to how costly it is to rent to a short term tenant. The mass of landlords and the mass of tenants actively participating in the search market in a given period is endogenous and determined by how many apartments become vacant and by how many tenants did not manage to find an apartment in the previous periods.

The effect of tenants paying the fee is that it serves to screen short term tenants from long term tenants, because a one-time fee has a greater effect on the per period expenses of a short term tenant than on the per period expenses of a long term tenant. If the landlord – rather than the tenant – pays the fee, screening is no longer possible.

This highlights a welfare reducing effect of banning tenant fees: screening is not feasible, so that landlords with high costs may choose to leave their apartment vacant as long as their costs stay high rather than rent their apartments out, which would lead to the risk of being matched with a short term tenant. However, there is also a more subtle welfare increasing effect of the landlords paying the fee, which only appears in

[1] In theory, the law prescribes that the party who hires the real estate broker should pay the fees. In practice, the law effectively bans fees being charged to tenants and requires landlords to pay the fees. While in theory, a broker could be hired by a prospective tenant, so that the tenant would have to pay the fee, this is not feasible in practice. The reason is that if a tenant were to hire a broker, then the tenant would have to have an exclusive agreement for an apartment, i.e., the broker would not be allowed to show that apartment to other tenants. This makes it unattractive for brokers to be hired by tenants. For details, see Michaelis and von Wangenheim (2016).

[2] See Bar-Isaac and Gavazza (2015).

a dynamic search environment. Landlords paying the fee leads to a higher turnover of apartments, since a lower fraction of apartments are occupied by long term tenants. This can mean that there is a vibrant search market with a large number of low cost landlords and short term tenants, which increases efficiency, due to the higher probability that a match between a low cost landlord and a short term tenant occurs.

Our results have a number of policy implications, besides the trade-off concerning banning tenant fees described above. Our results suggest that banning tenant fees comes at the cost of more vacancies (but higher turnover) of apartments. Our results also imply that if landlords rather than tenants pay the fees, rents will increase for two reasons. The first, more obvious reason, is that landlords will pass on the fees they have to pay to the tenants. There is, however, an additional effect: in a separating equilibrium in which landlords screen tenants, landlords have to give tenants informational rents. Once a separating equilibrium is not possible, landlords do not need to pay informational rents and apartment rents increase. This can lead to the ironic effect that a policy introduced to improve tenants' utilities may actually lead to a lower *net* utility for tenants (i.e. a lower utility from renting even after subtracting fees).

These results on rents may provide a possible explanation why rent control laws introduced in Germany simultaneously with the ban on tenant fees have been found ineffective empirically.[3] It also provides a possible explanation why empirical studies for other markets have found that if the landlord pays the fee, rents are higher[4] and vacancies are higher.[5]

Related literature This paper is most closely related to the search and dynamic random matching literature, which includes Wolinsky (1988), Satterthwaite and Shneyerov (2007, 2008), Shneyerov and Wong (2010), Lauermann et al. (2012), Lauermann (2013), Niedermayer and Shneyerov (2014), Lauermann and Wolinsky (2016). These papers put emphasis on the dynamics of markets and on the fact that the mass of traders active in

[3]See Kholodilin et al. (2016). However, since the policy has been in effect for only a year, it might be too early to draw a definite conclusion.
[4]See Ben-Shahar (2001).
[5]See Bar-Isaac and Gavazza (2015).

a search market is an endogenous outcome of the market equilibrium. The key difference is that while sellers typically do not care who buys their good, landlords do care who rents their apartment. Therefore, they screen different types of tenants, which makes the main question of this paper relevant: what the effect is of whether the tenant or the landlord pays the fee. Another key difference is that we endogenize the inflow of landlords: landlords only enter the search market if their apartment becomes vacant, which depends on how frequently landlords are matched with short term or long term tenants.

This paper also relates to the theoretical and empirical literature on brokerage fees on real estate *sales* (see Hsieh and Moretti (2003), Loertscher and Niedermayer (2016a,b); see also Han and Strange (2014, p. 850ff) for an overview of the literature on real estate brokers). Again, the main difference is that of sales versus rental, with the latter making screening and the question who pays the fees relevant.[6] Ben-Shahar (2001) empirically investigates the effect of brokerage cost allocations and Michaelis and von Wangenheim (2016) describe the legal situation surrounding the German Appointment Principle.[7] More generally, our paper relates to the literature on real estate brokers and real estate (see Han and Strange (2014) and the references therein).

2 Model Setting

We consider an infinite horizon steady-state rental market. Periods are denoted by $t = ..., -2, -1, 0, 1, 2, ...$ The market can be best thought of as a city with a fixed

[6] A further difference to Loertscher and Niedermayer (2016a) is that here we assume brokers' fees to be exogenous (which can be viewed as fees that cover brokerage costs). Loertscher and Niedermayer (2016a) consider profit maximizing real estate brokers and derive the optimal price schedule. This paper abstracts away from the fairly complex issues of optimal price schedules by brokers in order to be able to focus on the screening problem and the dynamic random matching equilibrium of the market. Loertscher and Niedermayer (2016b) consider the effect of different pricing schemes of intermediaries (in sales markets) on the possible preemption of a bilateral exchange.

[7] Michaelis and von Wangenheim (2016) contains an informal argument of the screening effect of tenants' fees. The empirical study in Ben-Shahar (2001) is supplemented by a reduced form theoretical model, which assumes that an increase of the fee paid by the landlord directly causes an increase of the vacancy duration after a tenant moves out. Besides formalizing the screening argument, we also depart from these papers by embedding the screening problem in a dynamic random matching model, which provides novel insights.

number of apartments available and a constant inflow and outflow of potential tenants.

□ Landlords. There is a continuum of landlords with total mass S^0, each endowed with one apartment. The landlords set per-period rents p. Once a tenant moves out, a landlord incurs a moving-out or inconvenience cost c. Landlords are heterogeneous with respect to c. At the beginning of each period, each landlord receives an i.i.d. shock: with probability μ_1 his next period moving-out cost is c_1, and with probability $\mu_2 = 1 - \mu_1$ his next period moving-out cost is $c_2 > c_1$. The probability distributions of the shocks are also assumed to be i.i.d. across periods. The i.i.d. cost may also reflect the uncertainty about landlords' outside options (e.g., an unexpected housing price boom may lead landlords to sell their apartments). Landlords get to know their next period costs c at the beginning of a period.

□ Tenants. In each period, a group of new tenants with total mass equal to 1 enter the market. A tenant's per-period gross value from renting an apartment is v. In any period t, all tenants, including the existing ones and the newly entered ones, receive an i.i.d. shock regarding when they leave the market. With probability λ_f, they stay only for one more period (period t) and will leave the market for good. We call the tenants who newly enter the market but will only stay for one period short term tenants (the subscript f represents "*fly-by-night tenants*"). After leaving the market, a tenant's utility is 0. With probability $\lambda_l = 1 - \lambda_f$, a tenant will stay in the market at least until the end of period $t + 1$. But in period $t + 1$, they will receive another i.i.d. shock regarding whether they will leave the market in the end of period $t + 2$. We call the tenants who will stay at least for the current period and the next period long term tenants.

□ Matching process. There is a competitive market of brokers who match landlords and tenants. The cost of helping landlords to rent out an apartment is $k < v$ and the fee per match is at the competitive level k. Without an intervention, fees are paid by tenants who successively rent apartments. If the Appointment Principle is imposed, landlords pay the fees.

There is a search market for the tenants who are unmatched from previous periods but have not yet left the market, the newly entered tenants, and the landlords with vacancies who are willing to rent out their apartments. Since we focus on the steady-state equilibria, the measure of each type of agent is constant across periods. Let μ be the fraction of unmatched landlords who are willing to participate, i.e., $\mu = 0$ if no landlords participate, $\mu = \mu_1$ if only landlords with cost c_1 participate, $\mu = 1$ if both types of landlords participate. We assume the matching function takes a simple form such that the total number of matches is equal to the short side of the matching pool, $M = \min\{B, S\mu\}$, where B is the measure of active tenants in the search market, S is the measure of available apartments, $S\mu$ is the measure of active landlords in the search market (that is the measure of landlords who have a free apartment and are also willing to rent it out). The matching probability for a tenant is $m_B = \frac{M}{B}$, while the matching probability for a landlord is $m_S = \frac{M}{S\mu}$.

All agents have the discount factor $\beta \in (0, 1)$. We impose the following assumption which makes sure that screening occurs in equilibrium if possible and that some landlords are unwilling to rent out if they cannot screen. This allows us to focus on the screening problem without having to go through too many cases.

Assumption 1. *The following conditions hold: (i)* $c_1 < \frac{v-k}{\beta}$*; (ii)* $\frac{v-k(1-\beta\lambda_l)}{\beta\lambda_f} < c_2$*; (iii)* $c_2 < \frac{1+\beta\lambda_f\mu_f}{\beta\lambda_f\mu_f}\frac{v-k}{\beta}$.

Assumption 1 (i) means that a trade between a short term tenant and a low cost landlord generates positive surplus. Assumption 1 (ii) implies repeatedly trading with a landlord who always has high cost generates negative surplus. As it will become apparent later, Assumption 1 (iii) means that high cost landlords are willing to rent out their apartments if they can screen, because their costs are not too high.

We will focus on the case where landlords are on the short side of the search market, i.e., $S < B$, which is plausible for most rental markets. This can be ensured by the following assumption.

Assumption 2. *The mass of apartments is sufficiently small*

$$S^0 < \frac{1}{\lambda_f}.$$

Assumption 2 can be interpreted in the following way: if all apartments are rented out and fraction λ_f of tenants renting these apartments are short term, then mass $\lambda_f S_0$ of apartments become available in a period. This is assumed to be less than the mass of entering tenants.

Discussion of assumptions and notation. For the sake of concreteness, it is useful to think of the market as a city. A tenants' type changes from l to f e.g. because they may get a job offer in a different city. As usual in housing market models, we assume that potential tenants have some outside option (with a utility normalized to 0) in case they do not find an apartment. The usual assumption is that there is a perfectly elastic housing market in the countryside surrounding the city, so that tenants who do not find an apartment in the city have the disutility of longer commute times. Landlords' moving out costs may change e.g. because in some periods of time they are more busy with their jobs. Alternatively, a landlord may move out of the city, in which case the hassle of dealing with the apartment is larger.

The example of a city as a market should not be taken too literally. The market may also be e.g. the rental submarket for downtown condominiums. Once a couple has children, they move out of this submarket e.g. to a single-family house in a suburb.

We have chosen the notation f and l for "fly-by-night" and "long term" and B and S for tenant and landlord (which is due to the dynamic random matching literature's tradition of dealing with (B)uyers and (S)ellers as the two sides of the market). This avoids the notational clash between "(l)andlord" and "(l)ong term" and between "(s)hort term" and "(s)eller". The negative connotation of "fly-by-night operator" is not necessarily intended, it may be efficient from a social planner's point of view that also these short term tenants find an apartment.

3 Commission Paid by Tenants

We start by considering a setup in which the tenants have to pay the fee k upon moving into an apartment. The fee k has a different effect on a short term tenant and a long term tenant, since the short term tenant amortizes the fee over just one period, whereas the long term tenant amortizes it over multiple periods. A landlord can therefore pursue one of two possible strategies: set a low rent p_L that will be accepted even by short term tenants, or, alternatively, set a high rent p_H which will only be accepted by long term tenants. Long term tenants accept the high rent in order to avoid search frictions due to having to wait an additional period.

In the following, we focus on an equilibrium in which landlords with low costs c_1 set a low rent p_L and landlords with high costs c_2 set a high rent p_H. As we will see later on, this is indeed an equilibrium under Assumptions 1 and 2.

The low rent is clearly $p_L = v - k$, since it sets the short term tenant just indifferent between renting and not, so that a short term tenant's utility of participating in the market is $V_f = 0$.[8]

The high rent p_H is slightly more involved. Denote the net present value of a long term tenant's utility from accepting a rent of p_H as V_l. The tenant gets utility $v - p_H$ from staying in the apartment in any given period. The probability that the tenant stays in the apartment at least one period is 1. The probability that he stays in the apartment at least t periods is λ_l^{t-1}, i.e., it is the probability that he is not hit by a shock for $t-1$ periods. Hence, taking into account the discount factor β and the initial fee k, we get

$$V_l = (v - p_H)\left[1 + \sum_{\tau=1}^{\infty} \beta^\tau \lambda_l^{\tau-1}\right] - k = (v - p_H)\underbrace{\left[1 + \frac{\beta}{1 - \beta\lambda_l}\right]}_{=B_1} - k. \qquad (1)$$

In equilibrium, the offer made to a long term tenant is such that he is just indifferent between accepting and rejecting the offer, i.e.

$$V_l = \beta\lambda_f m_B V_f + \beta\lambda_l \left[m_B\mu_1\left((v - p_L)\left(\sum_{t=1}^{\infty}\beta^t\lambda_l^{t-1}\right) + v - p_L - k\right) + (1 - m_B + m_B\mu_2)V_l\right], \qquad (2)$$

where the expression including V_f represents the possibility that the tenant becomes short term in the next period. The expression in square brackets is for the case that he stays as a long term tenant and either meets a low-cost landlord (probability $m_B\mu_1$) or a high cost landlord (probability $m_B\mu_2$) or is not matched in the subsequent period

[8] The pricing problem is trivial in this setting as it follows the similar logic as in the Diamond Paradox – a setting in which the sellers can fully extract consumer surplus.

(probability $m_B\mu_2$). In the two latter cases the tenant has to wait again until the next period, either because he is unmatched or because he again rejects the offer.

Solving equation (2) for V_l, we get

$$V_l = \underbrace{\frac{m_B\beta^2\lambda_l\mu_1}{(1-\beta\lambda_l)(1-\beta\lambda_l(1-m_B\mu_1))}}_{=B_2} k$$

To get p_H, we plug V_l into (1) and solve for p_H, which yields

$$p_H = \frac{B_1 v - (B_2+1)k}{B_1}. \tag{3}$$

Next, we consider the payoffs of landlords. Denote the expected utility of a landlord as $\mathbb{E}[W(c)]$. We will first describe the individual components of $\mathbb{E}[W(c)]$, then write down the recursive relation that determines $\mathbb{E}[W(c)]$ in our dynamic setup, and finally solve for $\mathbb{E}[W(c)]$. Denote by $\mathbb{E}[c] = \mu_1 c_1 + \mu_2 c_2$ the expected moving out costs of a landlord.

The utility of a landlord with cost c who rents out his apartment to a short term tenant is

$$W_f(c) = p_L - k - \beta c + \beta\mathbb{E}[W(c)].$$

The utility of a landlord who rents out his apartment to a long term tenant with rent p_H is

$$\tilde{W}_l = p_H\left(1+\sum_{\tau=1}^{\infty}\beta^\tau\lambda_l^{\tau-1}\right) + (\mathbb{E}[W(c)]-\mathbb{E}[c])\left(\sum_{\tau=2}^{\infty}\lambda_f\lambda_l^{\tau-2}\beta^\tau\right),$$

where the expression including p_H is the net present value of the rent (renting out the apartment in periods t and $t+1$ for sure, and renting out the apartment in any subsequent period $t+\tau$ with the probability $\lambda_l^{\tau-1}$ that the tenant did not become short term in $\tau-1$ periods) and the expression including $\mathbb{E}[W(c)]-\mathbb{E}[c]$ represents the net present value of renting out the apartment after the tenant moved out minus the moving out costs.

The utility of renting out to a long term tenant at rent p_L, $\hat{W}_l$ is identical to $\tilde{W}_l$, except that the rent is p_L rather than p_H:

$$\hat{W}_l = p_L\left(1+\sum_{\tau=1}^{\infty}\beta^\tau\lambda_l^{\tau-1}\right) + (\mathbb{E}[W(c)]-\mathbb{E}[c])\left(\sum_{\tau=2}^{\infty}\lambda_f\lambda_l^{\tau-2}\beta^\tau\right),$$

Next, we can write the utility W_H of a landlord who charges p_H and rents out only to long term tenants. Observe that W_H is independent of the current realization of c, since long term tenants will not move out of the apartment in the subsequent period. We have

$$W_H = m_S \lambda_l \tilde{W}_l + (1 - m_S \lambda_f)\beta\mathbb{E}[W(c)],$$

where the first term is the utility of being matched with a long term tenant and the second term is the utility of not being matched with a long term tenant (either due to being matched with a short term tenant or due to not being matched at all).

A landlord with costs c that chooses to ask for a low rent p_L and who therefore rents out both to short term and to long term tenants either rents out to a short term tenant, rents out to a long term tenant, or does not meet any tenants and has to wait a period, so that his expected utility is

$$W_L(c) = \lambda_f m_S \hat{W}_f(c) + \lambda_l m_S \hat{W}_l + (1 - m_S)\beta\mathbb{E}[W(c)].$$

For the equilibrium in which low cost landlords set the low rent p_L and high cost landlords set the high rent p_H to exist, we need the following conditions to hold: (i) low cost landlords prefer asking a high rent to asking a low rent $W_L(c_1) \geq W_H$, (ii) low cost landlords prefer asking a low rent to staying out of the market for a period $W_L(c_1) \geq \beta\mathbb{E}[W(c)]$, (iii) high cost landlords prefer asking a high rent to asking a low rent $W_H \geq W_L(c_2)$, and (iv) high cost landlords prefer setting a high rent to staying out of the market for a period $W_H \geq \beta\mathbb{E}[W(c)]$. Note that conditions (i) and (iv) imply condition (ii), which can therefore be ignored.

In an equilibrium in which landlords are on the short side, it can be shown that a combination of conditions (i) and (iii)

$$W_L(c_2) \leq W_H \leq W_L(c_1)$$

is equivalent to

$$\beta c_2 \geq v - k \geq \beta c_1, \tag{4}$$

by plugging in the expressions for $W_L(c_1)$, W_H, and $W_L(c_1)$, and some algebra.

Condition (4) is quite intuitive: βc is the discounted cost of renting out to a short term tenant who will move out in the next period and cause costs c; $v-k$ represents the gains from trade from renting out to a short term tenant.

The condition $W_H \geq \beta\mathbb{E}[W(c)]$ is somewhat more involved, but can still be simplified to

$$v-k \geq \frac{\beta\lambda_f\mu_2}{1+\beta\lambda_f\mu_2}\beta c_2,$$

that is, the gains from trade $v-k$ are sufficiently large. Observe that the condition does not contradict condition (4), since the fraction on the right-hand side is less than one.

Putting the pieces together, the expected pre-match value for landlords is

$$\mathbb{E}[W(c)] = \mu_1 W_L(c_1) + \mu_2 W_H.$$

$\mathbb{E}[W(c)]$ can be obtained by plugging in $W_L(c_1)$ and W_H and solving the recursive relation for $\mathbb{E}[W(c)]$. For markets in which the landlords are on the short side ($m_S = 1$), the solution for the pre-match expected value for landlords simplifies to

$$\mathbb{E}[W(c)] = \frac{(v-k)[1+\beta\lambda_f^2\mu_2 - \lambda_f(1-\beta)\mu_2] - \beta\lambda_f[c_1\mu_1 + c_2\beta\lambda_l\mu_2]}{1-\beta}. \tag{5}$$

After dealing with tenants' and landlords' payoffs, we now turn to the inflow-outflow equilibrium conditions of the matching market. The mass of landlords active in the market today is the sum of (a) the landlords active in the previous period who had low costs and were matched with a short term tenant, who quit in the current period $\lambda_f\mu_1 S$, (b) the high cost landlords active in the previous period who were matched with a short term tenant and therefore left their apartment vacant $\lambda_f\mu_2 S$, (c) all landlords who were inactive in the previous period whose long term tenant became a short term tenant $\lambda_f(S^0 - S)$. Formally,

$$S = \lambda_f\mu_1 S + \lambda_f\mu_2 S + \lambda_f(S^0 - S),$$

which simplifies to $S = \lambda_f S^0$. The tenants actively searching today are those who did not get matched in the previous period $(1-m_B)B$ and those who entered in this period:

$$B = (1-m_B)B\lambda_l + 1.$$

Observe that the short term tenants who got matched with a high cost landlord in the previous period did not find an apartment, but are nonetheless not active in the current period, since they left the market. The above expression simplifies to $B = (B - S)\lambda_l + 1$ due to $m_B = S/B$.

Putting together the equation for S and the equation for B, we get $B = 1/\lambda_f - \lambda_l S^0$ and $m_B = S^0\lambda_f^2/(1 - S^0\lambda_l\lambda_f)$.

Proposition 1. *When tenants pay the commissions, the equilibrium is such that*

(a) the landlords with costs c_1 set a low rent $p_L = v - k$ and rent to both types of tenants, whereas the landlords with cost c_H set a high rent p_H according to (3) and only rent to long term tenants.

(b) the equilibrium participation of landlords and tenants in the search market are respectively given by

$$S^b = \lambda_f S^0 \quad \textit{and} \quad B^b = \frac{1}{\lambda_f} - \lambda_l S^0. \tag{6}$$

The superscript b denotes that these are equilibrium masses of landlords and tenants in the setup in which the tenant pays the fee.

4 Commission Paid by Landlords

Now suppose making tenants pay the fees is banned and landlords therefore have to pay the brokerage fees. This implies that all landlords who participate in the search market cannot screen tenants and will all set the same rents. Anticipating this, a tenant will accept any offer with $p \leq v$ and will reject any offer with $p > v$.

We next consider landlords' surplus from participating in the search market. When being matched with a short term tenant, the post-match value function for the landlord is

$$W_f(c) = p - k - \beta c + \beta\mathbb{E}[W(c)]. \tag{7}$$

This landlord gets rent p from the tenant and pays fee k. Since the tenant is short term, she will move out for sure in the next period, the landlord will incur moving cost c in the next period, and has the continuation value $\mathbb{E}[W(c)]$. When being matched with a long term tenant, the post-match value for the landlord is

$$W_l = \left(1 + \sum_{\tau=1}^{\infty} \lambda_l^{\tau-1}\beta^{\tau}\right) p - k + \sum_{\tau=2}^{\infty} \lambda_f \lambda_l^{\tau-2}\beta^{\tau}\left(\mathbb{E}[W(c)] - \mathbb{E}[c]\right) \tag{8}$$

Since the tenant is long term, she may stay in the apartment for more than one period, with probability λ_l continuing with the rental each period. Moreover, since the tenant will stay at least to the end of next period, the landlord's cost when the tenant moves out is $\mathbb{E}[c]$.

If a landlord does not participate in the search market, his expected payoff is just $\beta\mathbb{E}[W(c)]$. Suppose a landlord with moving cost c participates in the search market. With probability $\lambda_f m_S$ this landlord will be matched with a short term tenant; with probability $\lambda_l m_S$, he will be matched with a long term tenant; with probability $1 - m_S$ he will not be matched at all. The landlord's expected value is then given by

$$\begin{aligned} W(c) =& \max\left\{\lambda_f m_S W_f(c) + \lambda_l m_S W_l + (1 - m_S)\beta\mathbb{E}[W(c)], \beta\mathbb{E}[W(c)]\right\} \\ =& \beta\mathbb{E}[W(c)] + \max\left\{m_S\left(\lambda_f W_f(c) + \lambda_l W_l - \beta\mathbb{E}[W(c)]\right), 0\right\}. \end{aligned}$$

Plug in $W_f(c)$ and W_l, we get

$$W(c) = \beta\mathbb{E}[W(c)] + m_S \max\left\{A_1 + A_2\mathbb{E}[W(c)] + A_3 c, 0\right\}, \tag{9}$$

where $A_1 = \frac{p-k+\lambda_l\beta k-\lambda_f\lambda_l\beta^2\mathbb{E}(c)}{1-\lambda_l\beta}$, $A_2 = \frac{\lambda_l(\beta^2-\beta)}{1-\lambda_l\beta}$, and $A_3 = -\lambda_f\beta$.

Under Assumption 1, the following conditions can be shown to hold,

$$A_1 + A_2\mathbb{E}[W(c)] + A_3 c_1 \geq 0 \quad \text{and} \quad A_1 + A_2\mathbb{E}[W(c)] + A_3 c_2 < 0,$$

by plugging in A_1, A_2, and A_3 and rearranging for c_1 and c_2. This implies only low cost landlords participate in the search market but the high cost landlords do not. Moreover, since landlords are assumed to be on the short side, $m_S = 1$. Taking expectation of (9) over c, we get

$$\mathbb{E}[W(c)] = \beta\mathbb{E}[W(c)] + \mu_1\left(A_1 + A_2\mathbb{E}[W(c)] + A_3 c_1\right),$$

and thus,

$$\mathbb{E}[W(c)] = \frac{\mu\left(A_1 + A_3 c_1\right)}{1 - \beta - A_2\mu_1}.$$

In equilibrium, every landlord who participates in the search market sets $p = v$ as this is the maximum rent a tenant will accept. Plug in the expressions of A_1, A_2, and A_3. The expected surplus is

$$\mathbb{E}[W(c)] = \frac{\mu_1\{v - k(1-\beta\lambda_l) - \beta\lambda_f[c_1(1-\beta\lambda_l\mu_2) + c_2\beta\lambda_l\mu_2]\}}{(1-\beta)(1-\beta\lambda_l\mu_2)}. \tag{10}$$

Consider the transitions in the search market. On the tenant side, the fraction of short term tenants is always λ_f and the fraction of long term tenants is always λ_l. Among the entrants, the fractions are λ_f and λ_l for short term tenants and long term tenants respectively. Among the unmatched tenants who are still in the city, the short term ones are gone. The long term ones receive the shock with the same probability.

Since landlords are on the short side, they can always get matched through the search market if they are willing to participate. In period $t-1$, $B - S\mu$ tenants do not get matched, but only a fraction $1-\lambda_f$ of them still stay in the market in period t, while a fraction λ_f drop out. There are also new entrants with total mass equal to 1. Putting these together, the mass of tenants in the search market in period t is

$$B = (B - S\mu_1)(1-\lambda_f) + 1.$$

On the landlord side, all landlords who participated in the search market in $t-1$ got matched but some of them were matched with short term tenants ($S\mu\lambda_f$). Among landlords whose apartments were occupied in period $t-1$, their apartments become available in period t if their tenants become short term and leave ($(S^0 - S)\lambda_f$). Also, $S(1-\mu)$ landlords who had cost c_2 in period $t-1$ did not participate, and are therefore still available. To sum up,

$$S = S(1-\mu_1) + (S^0 - S + S\mu_1)\lambda_f.$$

Combining the above equations, we obtain the steady-state masses of landlords and tenants that own available apartments, S^s and B^s, respectively. We state the steady-state results in the following Proposition.

Proposition 2. *When tenants pay the fees, the equilibrium is such that*

(a) the landlords with cost c_1 join the search market and set $p = v$ and rent to both types of tenants, whereas the landlords with cost c_2 do not participate.

(b) the equilibrium participation of landlords and tenants in the search market are

$$S^s = \frac{S^0 \lambda_f}{\lambda_f + \mu_1 - \lambda_f \mu_1} \quad \textit{and} \quad B^s = \frac{1 - \left(\frac{S^0 \lambda_f}{\lambda_f + \mu_1 - \lambda_f \mu_1}\right) \mu_1 \lambda_l}{\lambda_f}. \tag{11}$$

5 The Impact of the Appointment Principle

We first show which fee arrangement yields higher expected surplus for the landlords. When tenants pay the fee, the expected surplus of landlords is given by (5). When landlords pay the fee, the expected surplus of landlords is given by (10). The next proposition shows that the Appointment Principle is less preferred by landlords when $\frac{k}{v}$ and c_2 are small.

Proposition 3. *Banning tenants paying the fee reduces landlords' expected surplus if and only if*

$$\frac{k}{v} < \frac{[1 - \beta(1 - \lambda_f)][1 + \beta \lambda_f (1 - \mu_1)](1 - \mu_1)}{1 - \mu_1 - \beta[1 - 2\mu_1 - \lambda_f (1 - \mu_1)(2 - \beta(1 - \lambda_f)(1 - \mu_1) - \mu_1)]}.$$

and

$$c_2 < \frac{\left\{ \begin{array}{c} v\Big[[1-\beta(1-\lambda_f)][1+\beta\lambda_f(1-\mu_1)](1-\mu_1)) \\ -(\frac{k}{v}[1-\mu_1+\beta(-1+2\mu_1+\lambda_f(1-\mu_1)(2-\beta(1-\lambda_f)(1-\mu_1)-\mu_1))]\Big] \end{array} \right\}}{\beta^2 \lambda_f [1 - \beta(1 - \lambda_f)](1 - \lambda_f)^2}$$

Intuitively, when the trade value $v - k$ is large ($\frac{k}{v}$ small) or the moving cost c_2 is not that large, landlords benefit from tenants paying fees so that screening becomes feasible and more trades can be realized. The Appointment Principle rules out the feasibility of screening and thus lowers landlords' expected surplus.

We next turn to the comparison of total surplus under the two arrangements of fee payment. Given that we focus on steady-state equilibria, the total surplus is calculated as the total trade surplus generated in one period less the total cost incurred in that period.

Suppose that tenants pay the fees. In the following, we will go through all the components of the total surplus. First, consider the total moving out cost incurred in an arbitrary period t. Recall that in the corresponding steady-state equilibrium low cost landlords set a low rent to attract both types of tenants while high cost landlords set a high fee to only attract long term tenants. $\lambda_f \mu_1 S^b$ low cost landlords got matched with short term tenants in period $t-1$. Their tenants leave the apartments in period t and each of these landlords incur cost c_1. $\lambda_f \mu_2 S^b$ low cost tenants got matched with long term tenant in period $t-1$ and do not incur any cost in period t. $\lambda_f(S^0 - S^b)$ landlords did not participate in the last search market but their tenants leave the apartments in period t. Since these landlords did not participate in the last search market, their expected moving cost is $\mathbb{E}(c)$.

Next consider the surplus generated in period t. $\mu_1 S^b + \mu_2 \lambda_l S^b$ low cost landlords get matched in the current period and the generated surplus per apartment is $v - k$. For the $S^0 - S^b$ apartments which are unavailable for renting in the current period, each of them generates surplus v. In sum, the total surplus when tenants pay the commissions is

$$TS^b = (\mu_1 + \mu_2\lambda_l)S^b(v-k) + (S^0 - S^b)v - \lambda_f \mu_1 S^b c_1 - \lambda_f(S^0 - S^b)(\mu_1 c_1 + \mu_2 c_2). \quad (12)$$

Now suppose landlords pay the fees. Among the landlords who have their tenants move out in period t, $\lambda_f(S^0 - S^s)$ of them did not participate in period $t-1$ and their tenants become short term and move out in the current period. Each of these landlords has expected moving out cost $\mathbb{E}(c)$. $\mu_1 \lambda_f S^s$ landlords participated in the search market in period $t-1$ but their tenants move out in this period. These landlords must have moving cost c_1.

For the generated surplus, $\mu_1 S^s$ landlords get matched through the matching pool in period t and each of them generates surplus $v - k$. $S^0 - S^s$ landlords got matched previously and their tenants still stay in the current period. Each of these landlords generates surplus v. In sum, the total surplus when landlords pay the fees is

$$TS^s = \mu_1 S^s(v-k) + (S^0 - S^s)v - \lambda_f(S^0 - S^s)(\mu_1 c_1 + \mu_2 c_2) - \mu_1 \lambda_f S^s c_2. \quad (13)$$

We now compare TS^b and TS^s. The difference between TS^b and TS^s is

$$\begin{aligned} TS^b - TS^s &= (S^b - S^s)[\lambda_f \mu_2 c_2 + \mu_1(v-k) - v] + \mu_2 \lambda_l S^b (v-k) \\ &= \underbrace{[(S^0 - S^b) - (S^0 - S^s)][v - \mu_1(v-k) - \lambda_f \mu_2 c_2]}_{\text{turnover effect}} + \underbrace{\mu_2 \lambda_l S^b (v-k)}_{\text{screening effect}}. \end{aligned} \tag{14}$$

Note

$$S^b = \lambda_f S^0 \quad \text{and} \quad S^s = \frac{S^0 \lambda_f}{\lambda_f + \mu_1 - \lambda_f \mu_1}.$$

Therefore, $(S^0 - S^b) - (S^0 - S^s) > 0$. The Appointment Principle lowers efficiency if and only (14) is positive.

Proposition 4. *Appointment Principle reduces total welfare if and only if*

$$c_2 < \frac{\mu_1 k + \mu_2 v}{\lambda_f \mu_2}. \tag{15}$$

The Appointment Principle has two effects on total surplus. The first effect is the *screening effect*. Under the Appointment Principle, all high cost landlords with available apartments do not participate in the search market. Therefore, there is a forgone surplus: high cost landlords and long term tenants could trade with each other if tenants were to pay the fee. This is due to the fact that the Appointment Principle rules out the possibility of screening tenants. Note that the screening effect is also present in a static setting without dynamic random matching, search, and turnover. However, the second effect is unique to the dynamic random matching environment we consider here. We call it the *turnover effect*. Under the Appointment Principle, the number of occupied apartments in each period is smaller, as $(S^0 - S^b) > (S^0 - S^s)$. Although it leads to a loss of value generated from occupied apartments ($v - \mu_1(v-k)$ per occupied apartment), it also leads to a higher turnover rate ($S^s > S^b$). A higher turnover rate can mean that there is a vibrant search market with low cost landlords and short term tenants, which increases efficiency, due to the higher probability that a match between a low cost landlord and a short term tenant occurs ($\lambda_f \mu_2 c_2$ which is due to the fact that the high turnover reduces the number of cases in which a high cost landlord loses his tenant and incurs the cost c_2). When c_2 is large, the Appointment Principle may even enhance efficiency because of the turnover effect.

Figure 1 shows this possibility. Suppose $\lambda_f = \frac{1}{3}$, $k = 0.1$ $c_1 = 0$, $\beta = 0.9$, $\mu_1 = \frac{1}{3}$ and $S^0 = 1$. The horizontal axis is v while the vertical axis is c_2.

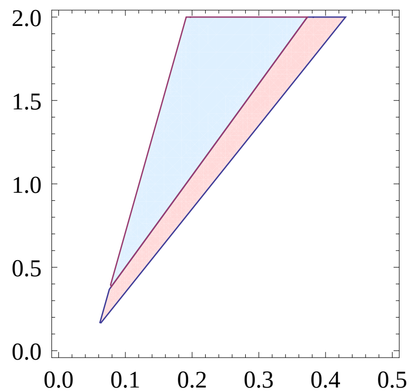

Figure 1: Efficiency comparison. Red area: tenant commission more efficient. Blue area: landlord commission more efficient. (Horizontal axis: v, vertical axis: c_2)

In Figure 1, the red region represents the parameter range in which tenants paying the commission is more efficient, and the blue region represents the parameter range where landlords paying the commission is more efficient. The figure shows that, when c_2 is large, it is more efficient to let landlords pay fees. The white regions in Figure 1 represent parameter ranges for which Assumption 1 is not satisfied.

We can modify condition (15) to get a lower bound for v above which the Appointment Principle reduces welfare

$$v > \frac{c_2 \lambda_f \mu_2 - \mu_1 k}{\mu_2}.$$

As in Figure 1, Figure 2 illustrates when the Appointment Principle may enhance efficiency. Suppose $\lambda_f = \frac{1}{3}$, $c_1 = 0$, $c_2 = 1$, $\beta = 0.9$, $\mu_1 = \frac{1}{3}$ and $S^0 = 1$. The horizontal axis is v while the vertical axis is k.

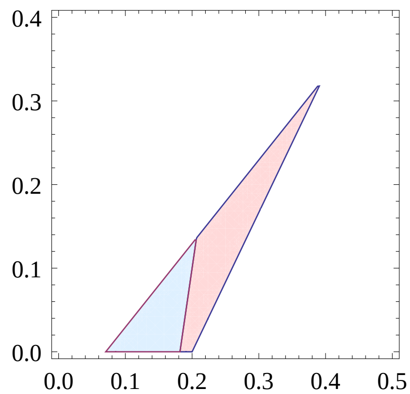

Figure 2: Efficiency comparison. Red area: tenant commission more efficient. Blue area: landlord commission more efficient. (Horizontal axis: v, vertical axis: k)

When v is large, it is more efficient to let tenants pay the fee, the reason being that allowing more landlords to participate in the search market generates greater surplus when v is large.

Condition (15) can also be modified to

$$\mu_1 > \frac{c_2 \lambda_f \mu_2 - \mu_2 v}{k}.$$

Suppose $v = 0.3$, $k = 0.1$ $c_1 = 0$, $c_2 = 1$, $\beta = 0.9$ and $S^0 = 1$. In the graph below, the horizontal axis is λ_f while the vertical axis is μ_1.

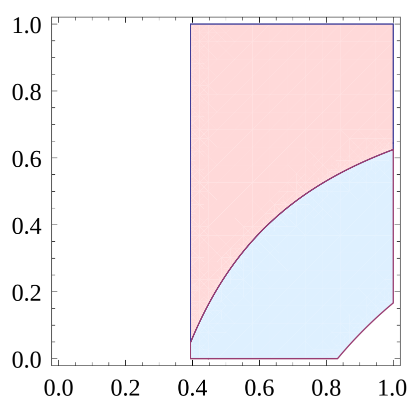

Figure 3: Efficiency comparison. Red area: tenant commission more efficient. Blue area: landlord commission more efficient. (Horizontal axis: λ_f, vertical axis: μ_1)

When μ_1 is small and therefore μ_2 is large, it is more efficient to let the landlord pay the fees. This is because landlords who have rented their apartments to tenants for more than one period are likely to incur the high cost c_2 when the tenants move out and a fast turnover can mitigate the problem.

6 Discussion

6.1 Policy Implications

With our results at hand, we can discuss the effects of a ban of tenant fees and of other policies that have been implemented or recommended in rental markets.

□ Vacancies. When landlords pay the fee, there are more available apartments in each period, compared to when tenants pay the fee, $S^b = S^0\lambda_f < S^0\lambda_f/(\lambda_f + \lambda_l\mu_1) = S^s$, i.e. there is a higher turnover of apartments. However, when landlords pay the fee, the high cost landlords who have available apartments are unwilling to join the search market. So the number of apartments offered when landlords pay the fee is $S^s\mu_1 = S^0\lambda_f\mu_1/(\mu_1 + \lambda_f\mu_2)$, which is lower than $S^b = S^0\lambda_f$. The vacancy rate (i.e. the fraction of apartments that are not rented out at the end of the period, i.e. the fraction of empty apartments) increases as a result of a ban of tenant fees, since $\mu_2 S^s/S^0 = \mu_2\lambda_f/(\lambda_f + \mu_1\lambda_l) > \mu_2\lambda_f^2 = \mu_2\lambda_f S^b/S^0$. The German real estate brokers' association reported a sharp decrease of apartments offered (not only through real estate brokers, but in general through online platforms) since June 2015, which is in line with the prediction of our theory.[9]

□ Rent control. Along with the Appointment Principle, a change of federal law in Germany permitted local governments to introduce rent controls (called rent brakes or "Mietpreisbremse"), which prescribe that after a tenant moves out, rents may be increased to at most 110% of typical rents in that neighborhood.[10] 308 local governments chose to introduce rent controls, which affects 20.7 million Germans or roughly one quarter of the population.[11] A study by the German Institute for Economic Research (Kholodilin et al. (2016)) claims that rent controls did not work, since rents increased

[9]For the number of apartments offered, see `http://www.spiegel.de/wirtschaft/service/makler-bestellerprinzip-wirkt-a-1094063.html`. Unfortunately, it is too early to have reliable data on vacancy rates.

[10]Obviously, there is some ambiguity in the law about what a typical rent means.

[11]See Kholodilin et al. (2016).

significantly after the introduction of the law. While it is hard to disentangle different effects on rents (after all, rents have been increasing for quite some time), it is worthwhile to point out that our theory would predict that rents should increase after a ban of tenant fees for two reasons: landlords will pass through fee to tenants; without the possibility of screening, long term tenants will not get an informational rent. More formally, assume that there is a rent cap of $\overline{p}$, which is "the typical rent in the neighborhood". If the rent cap is above the high prices ($\overline{p} > p_H$), we would expect all rents to rise to $\overline{p}$ after the reform. If the rent cap is between the high and the low prices ($p_L < \overline{p} < p_H$), then we should expect some rents to increase and some to decrease after the reform.

□ Vacancy tax. A remedy sometimes proposed to remedy the shortage of rental apartments is a vacancy tax. For example, the mayor of Vancouver (British Columbia) proposed a vacancy tax in September 2016.[12] The effect of a vacancy tax can be seen as an increase of v, i.e. an increase of the gains from trade for a rent. If the tax is high enough, then all landlords with available apartments will join the matching pool. This is likely to enhance efficiency when v or μ_1 is large. However, when c_2 is large, imposing taxes on vacancies might reduce efficiency as now even high-cost landlords need to rent to short term tenants which is socially inefficient.

6.2 Empirical Predictions

Our analysis has focused on one of two possible setups being exogenously given: the tenant pays the fee or the landlord pays the fee. One may wonder what our theory can teach us about markets in which market participants choose whether the tenant or the landlord should pay the fee. First, it should be noted that our theory does not directly apply to such a question, since we have not dealt with the endogenous choice of the setup. One could take different approaches of how to deal with such an endogeneity. For example, the setup might be chosen which generates the highest surplus for landlords. Or the setup that generates the highest total surplus.

[12] See http://www.cbc.ca/news/canada/british-columbia/vancouver-vacant-home-tax-1.3761496. It should be noted though that the proposal includes several qualifications, e.g. it excludes property owners who live a few months a year in Vancouver.

Keeping this caveat in mind, we can relate our results to the empirical observations about brokerage contracts in Manhattan described in Bar-Isaac and Gavazza (2015).

However, one can also interpret our results as applying to a setup in which the market participants choose one of the setups, e.g. the one which generates higher surplus for the landlords.

□ Moving-Out Costs c. While the landlord's costs of a tenant moving out are not directly observable, there is a proxy for them: if for a submarket of the rental market rents increase and rents are stabilized, the moving-out opportunity costs of landlords are lower. This is due to the fact that once a tenant moves out, landlords can increase rents. Therefore, in such submarkets, landlords worry less about screening long term versus short term tenant and landlord fees become more attractive for landlords. This is consistent with Bar-Isaac and Gavazza (2015)'s observation that in markets with stabilized rents, landlords are more likely to pay the broker's fee.

□ Vacancies. As stated before, in submarkets in which the landlord pays the fees, we should expect higher vacancy rates, since landlords with high-costs may not be willing to rent out there apartments if they cannot screen out short term tenants. Bar-Isaac and Gavazza (2015) find that there are more vacancies in submarkets in which the landlord pays the fee.[13]

6.3 Discussion of Assumptions and Possible Generalizations

□ Non-stationary rents. We have assumed that rental contracts consist of rents that are constant over time. However, if one were to allow rents to decrease over time, then even if tenant fees are banned, they could still be replicated by rents that decrease over time. The simplest form of replication would be simply a rent $p+k$ in the first period and a rent p in subsequent periods. One reason why we do not see such contractual arrangements in practice may be that there may be a hold-up problem: a landlord might charge a high rent initially, but refuse to lower the rent later on, claiming the the apartment needed

[13] Of course, causality may go in either direction between vacancies and who pays the fee. More vacancies may also weaken the landlord's bargaining position (in a model in which there is bargaining about the rent), so that the landlord may be forced to pay the broker's fee.

repairs.[14] It is also worthwhile noting that Genesove (2003) finds nominal rigidities in apartment rents, which may serve as a further justification for our assumptions that rents are constant over time.

☐ Full pooling. Under our assumptions, if tenant fees are banned, landlords with high costs stay out of the market. However, if c_2 is sufficiently low, landlords with high costs may participate even if tenant fees are banned. They may simply bite the bullet and take the risk that they rent out their apartment to a short term tenant. They are willing to do this if this risk is outweighed by the possibility of the tenant being long term. If high cost landlords are willing to participate, then a ban on tenant fees and hence the screening of tenants would still impose a cost for landlords: high cost landlords could not avoid the costs of renting out to short term tenants.[15]

☐ Persistence of types. We have assumed that both landlords' and tenants' types are essentially a new draw in every period. This greatly simplifies the analysis and avoids the well known challenges of persistent types that the literature on dynamic random matching has highlighted (see e.g. Satterthwaite and Shneyerov (2007, 2008)). We should expect the screening effect to carry over to a setup with persistent types and the turnover effect to carry over as long as there is a possibility of landlords' types changing over time.

☐ Labor market application. Our model can be readily extended to analyze other frictional markets such as labor markets. If a labor market is populated by a constant mass of potential employers and an inflow of workers who have either a high or a low quitting rate, then our model can shed light on how this market evolves as well. A fee paid to a labor market intermediary (such as a headhunter or a temporary employment intermediary) has a similar effect as the rental fees in our model.

[14] Such a hold-up is in principle also possible for rents that are constant over time: the landlord might charge a low rent initially and then excessively increase it later on. However, there is regulation directed at rent increases that restricts such actions.

[15] It is also possible that c_2 is so low that high cost landlords make positive profits if they rent out to short term tenants. In this case, even if tenant fees are allowed, high cost landlords would choose not to screen. In this case, it would be irrelevant whether fees are paid by the landlord or the tenant.

7 Conclusions

We have developed an equilibrium search model of rental markets. Similarly to e.g. Wolinsky (1988), Satterthwaite and Shneyerov (2007, 2008), we have modeled the mass of tenants entering the market as exogenous and the mass of tenants actively searching in the market as endogenous. We have departed from the literature in an aspect which is special to rental markets: that the mass of apartment (and landlords) is constant over time. Therefore, the mass of landlords entering the search market is endogenously given by the rate at which existing apartments become free.

We have investigated the effect of who pays the brokerage fee. Fees paid by tenants serve as a device to screen long term tenants from short term tenants, since a one-time fee is more costly per period to short term tenants. We have found a trade-off: switching to a regime in which landlords pay the fees comes at the cost that screening is no longer feasible, so that high cost landlords temporarily withdraw from the market and leave their apartments vacant. But the switch also comes with an advantage: it increases turnover and hence the matching efficiency, in particular, matches between low cost landlords and short term tenants become more likely.

Our theory also makes a number of empirically testable predictions on how tenant versus landlord fees relate to rent increases and vacancies. These predictions are consistent with empirical observations, even if it is too early to call, especially for the German rental market where the policy change is quite recent and where data availability should improve in the future.

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