Online Supplement for "Percentage Fees in Thin Markets: An Optimal Pricing Perspective" by Simon Loertscher and Andras Niedermayer

D Extension: Non-Stationarity

Assume now that the environment is described by known sequences $\boldsymbol{\delta} = (\delta_t)_{t=0}^{\infty}$ and $\boldsymbol{F} = (F_t)_{t=0}^{\infty}$, where δ_t is the discount factor in period t and F_t is the distribution from which buyers' types are drawn in period t and that for all t, $\Phi_t(v) = v - \frac{1-F_t(v)}{f_t(v)}$ is monotone in v. One example is the exponential discounting (or constant drop out probability) $\delta_\tau = \delta$ considered so far. Another is exponential discounting up to a deadline T after which the seller leaves the market for sure ($\delta_\tau = \delta$ for $\tau \leq T$ and $\delta_\tau = 0$ for $\tau > T$). Let π_B^t describe the arrival process of buyers in period t and denote by $F_{(1),t}(v)$ and $F_{(2),t}(v)$ the distributions of the highest and second-highest draw in t. Expected revenue given reserve p in period t, conditional on trade in period t, is then given as $R_t(p) = \frac{\int_p^{\overline{v}} \Phi_t(v) dF_{(1),t}(v)}{1-F_{(1),t}(p)}$.

Given a sequence $\boldsymbol{k}$ of expected transaction prices conditional on trade, the seller's ultimate probability of selling is still given as $\sum_{t=0}^{\infty} q_t(\boldsymbol{k})$, where

$$q_t(\boldsymbol{k}) := \left(1 - F_{(1),t}(R_t^{-1}(k_t))\right) \prod_{\tau=0}^{t-1} \delta_\tau F_{(1),\tau}(R_\tau^{-1}(k_\tau)).$$

Next define $1-\overline{F}(k) := \lim_{T\to\infty} 1-\overline{F}_T(k)$, where $1-\overline{F}_T(k)$ is the maximum of $\sum_{t=1}^{T} q_t(\boldsymbol{k})$ subject to the constraint $(\sum_{t=0}^{T} q_t(\boldsymbol{k}) k_t)/(\sum_{t=0}^{T} q_t(\boldsymbol{k})) = k$, as defined in the proof of Proposition 2. At date 0, the objective function that accounts for incentive compatibility provided the pointwise maximizer $k(c)$ of the integrand is monotone is then still given by (24), yielding the allocation rule allocation rule (25). Consequently, the functional form of the expectational fees $\overline{\omega}(k)$ under non-stationarity will be the same as under stationarity. Hence, it is as given in Lemma 2 in the proof of Proposition 3. This also implies that in the limit, as G converges to a mirrored Generalized Pareto distribution, the optimal expectational fee will be linear as in the stationary case.

Although $\omega_t(p)$ will in general vary over time because the environment is non-stationary, the linearity of the expectational fees $\overline{\omega}$ in the limit implies that the optimal transaction fees will be linear in the limit too.

E Common Value Component

In this appendix, we extend our model to account for common values between sellers and buyers. Our analysis rests on the linear, additively separable specification of Cai, Riley, and Ye (2007, Section V) and Jullien and Mariotti (2006).

E.1 Setup

One-Shot Model As in the independent private values model, the seller's type c is the seller's opportunity cost of selling (in the static model, that is with $\delta = 0$) or his cost of production, with the distribution of c being $G(c)$ with support $[\underline{c}, \overline{c}]$, density $g(c) > 0$ for all $c \in (\underline{c}, \overline{c})$. A buyer's type is now denoted x, where x is assumed to be distributed according to $F(x)$ with support $[\underline{v}, \overline{v}]$ and density $f(x) > 0$ for all $x \in (\underline{v}, \overline{v})$. All types are assumed to be independently distributed. The willingness to pay $v(x, c)$ of a buyer of type x who buys from a seller of type c is given as

$$v(x, c) := \lambda c + (1 - \lambda)x, \tag{32}$$

where $\lambda \in [0, 1]$ measures the severity of common-value component, with $\lambda = 0$ corresponding to the private values model and $\lambda = 1$ representing the pure common value model.

Dynamic Model with Percentage Fees The model we analyze is a dynamic version of the linear model sketched above and analyzed by Cai, Riley, and Ye (2007), which we augment by percentage fees.[94] The dynamic environment is the same as in the main body of the paper: In every period a random number of buyers arrive, draw their types

[94] A similar static model without percentage is analyzed by Jullien and Mariotti (2006) for the case of two buyers.

x_b independently from F with support $[\underline{x}, \overline{x}]$, and we assume that the realized number of buyers is not known by the seller when he sets the reserve.[95] We impose stationarity and assume that the common discount factor is $\delta \in [0, 1)$. Moreover, buyers do not condition their behavior on past reserve prices set by the seller. A simple assumption that implies this restriction is that they do not observe these prices.[96] As in the main text, we assume that the function $\Phi(x) := x - \frac{1-F(x)}{f(x)}$ is increasing.

Equilibrium Next we derive the separating equilibrium in the dynamic model with percentage fees $\omega(p) = bp$ with $b \in [0, 1)$. Let x be the type of a buyer who is indifferent between buying and not buying when the price is $\lambda\hat{c} + (1 - \lambda)x$ and the seller is believed to be of type $\hat{c}$. Denote by $W_S^{cv}(c, \hat{c}, x)$ the discounted expected payoff of a seller in the common-value setup when the seller's type is c, buyers believe that his type is $\hat{c}$ and when the indifferent buyer-type is x. Given the belief that the seller is of type $\hat{c}$, the willingness to pay of buyer of type x is $\lambda\hat{c} + (1 - \lambda)x$. Given x and $\hat{c}$, this will thus be the reserve price. Consequently, we have

$$\begin{aligned} W_S^{cv}(c, \hat{c}, x) &= \delta W_S^{cv}(c, \hat{c}, x) F_{(1)}(x) + [(1 - b)(\lambda\hat{c} + (1 - \lambda)x) - c][F_{(2)}(x) - F_{(1)}(x)] \\ &+ \int_x^{\overline{x}} [(1 - b)(\lambda\hat{c} + (1 - \lambda)y) - c] dF_{(2)}(y), \end{aligned}$$

where b is the proportional fee, that is $w(p) = bp$ with $b < 1$. The first summand is the expected discounted continuation value if no sale occurs today. The other two summands capture the expected payoff from a sale in the current period, in which case the game ends. Solving for $W_S^{cv}(c, \hat{c}, x)$ we get

$$\begin{aligned} W_S^{cv}(c, \hat{c}, x) &= \frac{1}{1 - \delta F_{(1)}(x)} \Big\{ [(1 - b)(\lambda\hat{c} + (1 - \lambda)x) - c][F_{(2)}(x) - F_{(1)}(x)] \\ &+ \int_x^{\overline{v}} [(1 - b)(\lambda\hat{c} + (1 - \lambda)y) - c] dF_{(2)}(y). \Big\}. \end{aligned}$$

[95] In contrast, Cai, Riley, and Ye (2007), and Jullien and Mariotti (2006), assume that the number of buyers is deterministic and commonly known.

[96] But even if they observed them, there would still be a perfect Bayesian equilibrium in which these prices are completely ignored on the equilibrium path.

which can be written more compactly, using the notation $1 - F_\infty(y) = \frac{1-F_{(1)}(y)}{1-\delta F_{(1)}(y)}$ and $R(x) = \frac{x[F_{(2)}(x)-F_{(1)}(x)]+\int_x^{\overline{v}} y dF_{(2)}(y)}{1-F_{(1)}(x)}$ introduced in the main text, as

$$W_S^{cv}(c,\hat{c},x) = (1-b)(1-F_\infty(x))\left[(1-\lambda)R(x) + \lambda\hat{c} - \frac{c}{1-b}\right]. \quad (33)$$

Let $W_{Si}^{cv}(.,.,.)$ denote the first derivative of W_S^{cv} with respect to its ith argument and $W_{Sij}^{cv}(.,.,.)$ the cross partial, that is the derivative of $W_{Si}^{cv}(.,.,.)$ with respect to its jth argument. Observe that

$$W_{S2}^{cv}(c,\hat{c},x) = (1-b)\lambda(1-F_\infty(x)) > 0 \quad (34)$$

and

$$W_{S3}^{cv}(c,\hat{c},x) = -(1-b)f_\infty(x)\left[(1-\lambda)\tilde{\Phi}(x) - \lambda\hat{c} - \frac{c}{1-b}\right], \quad (35)$$

where $\tilde{\Phi}(x) := R(x) - \frac{1-F_\infty(x)}{f_\infty(x)}R'(x)$ as in the main text.

Notice that in equilibrium x will depend on $\hat{c}$, so that we can write $x = x(\hat{c})$.[97] A necessary condition for a separating equilibrium is

$$W_{S2}^{cv}(c,c,x(c)) + W_{S3}^{cv}(c,c,x(c))x'(c) = 0. \quad (36)$$

Equivalently, (36) can be expressed as

$$W_{S2}^{cv}(c(x),c(x),x)c'(x) + W_{S3}^{cv}(c(x),c(x),x) = 0, \quad (37)$$

where $c(x)$ is the inverse of $x(c)$. The initial condition is

$$W_{S3}^{cv}(\underline{c},\underline{c},x^*(\underline{c})) = 0.$$

The results in Cai, Riley, and Ye (2007) imply that a separating equilibrium exists and is unique if $W_{S31}^{cv} > 0$ and if the function

$$\beta(c,x) := -\frac{W_{S3}^{cv}(c,c,x)}{W_{S2}^{cv}(c,c,x)} = \frac{f_\infty(x)}{\lambda(1-F_\infty(x))}\left[(1-\lambda)\tilde{\Phi}(x) - \frac{1-\lambda(1-b)}{1-b}c\right] \quad (38)$$

[97]For the linear specification we have, $x(\hat{c}) = \hat{c}$.

intersects with 0 at most once and, if it does, from below as x increases.[98] Observe that the equality in (38) follows by substituting the expressions for $W^{cv}_{S2}(c,\hat{c},x)$ and $W^{cv}_{S3}(c,\hat{c},x)$ evaluated at $\hat{c}=c$.

We are now going to show that the conditions identified by Cai, Riley, and Ye (2007) are satisfied in our setup. To that end, notice first that

$$W^{cv}_{S31}(c,c,x) = f_\infty(x) > 0.$$

Second, the function $\beta(c,x)$ will change its sign at most once (from negative to positive) as x increases if $\tilde{\Phi}(x)$ is increasing. We are now going to show that monotonicity of $\Phi(x)$ implies monotonicity of $\tilde{\Phi}(x)$. To see this, observe first that $R'(x) = \frac{f_{(1)}(c)}{1-F_{(1)}(c)}[R(x) - \Phi(x)] > 0$ and that therefore

$$\tilde{\Phi}(x) = R(x)\left[1 - \frac{1-\delta F_{(1)}(x)}{1-\delta}\right] + \frac{1-\delta F_{(1)}(x)}{1-\delta}\Phi(x).$$

Consequently,

$$\tilde{\Phi}'(x) = R'(x)\left[1 - \frac{1-\delta F_{(1)}(x)}{1-\delta}\right] + \frac{\delta}{1-\delta}f_{(1)}(x)[R(x)-\Phi(x)] + \frac{1-\delta F_{(1)}(x)}{1-\delta}\Phi'(x) > 0,$$

which is positive because each summand is positive. We have thus verified that the conditions Cai, Riley, and Ye (2007) hold in our setup.

Proposition 6. *(i) There is a unique separating equilibrium in which buyers only condition on the current period reserve price.*

(ii) In this equilibrium, the marginal type $x^*(c)$ is given by the differential equation

$$-\frac{1-F_\infty(x^*(c))}{f_\infty(x^*(c))}\lambda + {x^*}'(c)\left\{(1-\lambda)\tilde{\Phi}(x^*(c)) - \left(\frac{1}{1-b} - \lambda\right)c\right\} = 0 \qquad (39)$$

with the initial condition given by

$$\tilde{\Phi}(x^*(\underline{c})) = \frac{1-(1-b)\lambda}{(1-b)(1-\lambda)}\underline{c}. \qquad (40)$$

[98] As mentioned, Cai, Riley, and Ye (2007) do not analyze a model with a stochastic number of buyers as we do. However, a careful reading of their paper reveals that only the distributions $F_{(1)}(x)$ and $F_{(2)}(x)$ of the highest and the second-highest buyer type matter for their analysis. Therefore, this analysis extends directly to the setup with a stochastic number of buyers. Moreover, because there are no informational externalities between buyers in our setup, the additional conditions identified by Lamy (2010) are not needed.

(iii) Equivalently, the inverse marginal type function $c^*(x)$ in this equilibrium is given by the differential equation

$$-c'(x)\frac{1-F_\infty(x^*(c))}{f_\infty(x^*(c))}\lambda + x^{*'}(c)\left\{(1-\lambda)\tilde{\Phi}(x^*(c)) - \left(\frac{1}{1-b}-\lambda\right)c(x)\right\} = 0$$

with the initial condition

$$\tilde{\Phi}(\underline{x}^*) = \frac{1-(1-b)\lambda}{(1-b)(1-\lambda)}\underline{c}.$$

Proof of Proposition 6. Part (i) follows from Theorem 1 in Cai, Riley, and Ye (2007), and part (iii) follows from part (ii). We are therefore left to prove part (ii).

Substituting the expressions for the derivatives from (34) and (35) into the first-order condition (36), dividing by $f_\infty(x^*(c))$ and making the substitution $\tilde{\Phi}(x) = R(x) - \frac{1-F_\infty(x)}{f_\infty(x)}R'(x)$ yields (39). Making the same substitutions, the initial condition $W^{cv}_{S3}(\underline{c}, \underline{c}, x^*(\underline{c})) = 0$ becomes (40). The arguments by Cai, Riley, and Ye (2007) can be applied with minimal adjustments to account for the percentage fee b that, under the given assumptions, the solution to the initial condition and the differential equation is unique and characterizes a profit maximum. □

An important difference between the private values model and the separating equilibrium of the model with a common value component is that conditional on the buyer's type x, the transaction price $\breve{p}$ will vary with the reserve price p if there is a common value component. We now derive the distribution of the transaction price $\breve{p}$ conditional on the reserve price p, which we denote $\breve{F}(\breve{p}|p)$.

Let

$$P(c) := \lambda c + (1-\lambda)x^*(c)$$

be the strictly increasing reserve price function in the separating equilibrium. Let $x(\breve{p}, p)$ be the type of a buyer who is indifferent between buying and not buying when the transaction price is $\breve{p}$ and the reserve price is p. This indifferent type is given by

$$x(\breve{p}, p) = \frac{\breve{p} - \lambda P^{-1}(p)}{1-\lambda}.$$

Let

$$x(p) := x(p, p)$$

be the buyer type who is indifferent between buying and not buying at the reserve p.

Conditional on a transaction occurring, the probability that the transaction price is equal to the reserve price is thus

$$\Pr(\breve{p} = p) = \frac{F_{(2)}(x(p)) - F_{(1)}(x(p))}{1 - F_{(1)}(x(p))}.$$

The probability that the transaction price $\breve{p}$ satisfies $\breve{p} \leq \breve{p}_0$ for some $\breve{p}_0 > p$ is

$$\Pr(\breve{p} \leq \breve{p}_0) = \frac{F_{(2)}(x(\breve{p}_0, p)) - F_{(2)}(x(p))}{1 - F_{(1)}(x(p))}.$$

The cumulative distributions of transaction prices $\breve{p}$ conditional on trade occurring given reserve p is thus given as

$$\breve{F}(\breve{p}|p) = \begin{cases} 0 & \text{if } \breve{p} < p \\ \frac{F_{(2)}(x(p)) - F_{(1)}(x(p))}{1 - F_{(1)}(x(p))} & \text{if } \breve{p} = p \\ \frac{F_{(2)}(x(\breve{p},p)) - F_{(1)}(x(p))}{1 - F_{(1)}(x(p))} & \text{if } \breve{p} \in (p, \overline{x}],) \\ 1 & \text{if } \breve{p} > \overline{x}. \end{cases}.$$

E.2 Estimation of Dynamic Model with Percentage Fees

E.3 Identification

We now show non-parametric identification in a model with a common value component, restricting our attention to identification without unobserved heterogeneity. Observe that with independent private values (see Section 3.4) we have shown non-parametric identification without making use of the transaction prices. The basic idea of the following identification argument is that we use additional information on transaction prices to identify the weight of the common value component λ. Informally, under independent private values ($\lambda = 0$), a higher reserve does not change the distribution of transaction prices, except for a truncation. In the presence of a common value component, the distribution changes beyond a truncation as the reserve price increases. We state this idea more formally in the following.

Let $1 - H_\infty(p)$ be the probability of ever selling given reserve price p, which is empirically observable.[99] Further, and denote by $1-\overline{H}(k)$ the probability of ever selling with a common value, given the average transaction price k. The maximization problem for a seller of type c is then equivalent to choosing k to maximize $((1-b)k-c)(1-\overline{H}(k))$. The first-order condition is $(1-b)\left(k-\frac{1-\overline{H}(k)}{\overline{h}(k)}\right)-c=0$, yielding

$$\overline{P}^{-1}(k) := (1-b)\left(k-\frac{1-\overline{H}(k)}{\overline{h}(k)}\right)$$

as the inverse of the optimal average transaction price rule.

Let $R(p)$ be the average transaction price in any period conditional on a transaction occurring in this period, given the reserve p. Then $k=R(p)$ and, under monotonicity, $p=R^{-1}(k)$. Note that $R(p)$ is identifiable from the joint distribution of the reserve prices and the transaction prices.

The probability $q_p(\breve{p})$ that the transaction price is larger than $\breve{p}$ for some $\breve{p}>p$ is then

$$q_p(\breve{p}) = 1-F_{(2)}\left(x(\breve{p},p)\right).$$

This probability $q_p(\breve{p})$ is observable. Inverting $q_p(\breve{p})=1-F_{(2)}\left(x(\breve{p},p)\right)$, we get

$$q_p^{-1}(q) = \lambda P^{-1}(p)+(1-\lambda)F_{(2)}^{-1}(1-q).$$

Evaluating $q_p^{-1}(q)$ at two different reserve prices p_1 and p_2 and taking differences yields

$$q_{p_2}^{-1}(q)-q_{p_1}^{-1}(q) = \lambda[P^{-1}(p_2)-P^{-1}(p_1)].$$

Therefore, λ is identified by

$$\lambda = \frac{q_{p_2}^{-1}(q)-q_{p_1}^{-1}(q)}{P^{-1}(p_2)-P^{-1}(p_1)}. \tag{41}$$

Using

$$\begin{aligned}
\overline{H}(k) &= H_\infty(R^{-1}(k)) \\
\overline{P}^{-1}(k) &= \left(k-\frac{1-\overline{H}(k)}{\overline{h}(k)}\right)(1-b) \\
P^{-1}(p) &= \overline{P}^{-1}(R(p)).
\end{aligned}$$

[99] H is non-parametrically identifiable through the relation between the reserve and whether a property was sold.

and plugging this into (41) gives us λ. With λ at hand, one can use the same approach as for independent private values (see Section 3.4) to show identification of F, G, δ, and τ.

E.4 Likelihood function

We now derive the likelihood function.

No Unobservable Heterogeneity Assume first that there is no unobservable heterogeneity. Let

$$g_p(p) := g(P^{-1}(p))[P^{-1}(p)]'$$

denote the density of reserve prices.

The probability mass function given reserve p for time on the market t with $t = 0, .., \infty$ is then given, if the object is sold, as

$$[\delta F_{(1)}(x(p))]^t (1 - F_{(1)}(x(p)))$$

and for objects that do not sell as

$$[\delta F_{(1)}(x(p))]^t (1 - \delta) F_{(1)}(x(p)).$$

Let $S = 1$ if the object ever sells and $S = 0$ if it never sells. Let $h_{tpS}(t, p, S)$ denote the likelihood of observing (t, p, S). This function is given as

$$h_{tpS}(t, p, S) = \begin{cases} [\delta F_{(1)}(x(p))]^t (1 - F_{(1)}(x(p))) g_p(p) & \text{if } S = 1 \\ [\delta F_{(1)}(x(p))]^t (1 - \delta) F_{(1)}(x(p)) g_p(p) & \text{if } S = 0 \end{cases} .$$

Next, consider transaction prices $\breve{p}$. Let $h_{\breve{p}}(\breve{p}|p, S)$ denote the likelihood of observing $\breve{p}$ given (p, S) and set $\breve{p} = 0$ if $S = 0$. Under the assumption of stationarity, $h_{\breve{p}}(\breve{p}|p, S)$ is independent of t. By the previous arguments,

$$h_{\breve{p}}(\breve{p}|p, S) = \begin{cases} \breve{f}(\breve{p}|p) & \text{if } S = 1 \\ \Delta(\breve{p}) & \text{if } S = 0 \end{cases} ,$$

where $\Delta(.)$ is the Dirac delta-function and where the density $\breve{f}(\breve{p}|p)$ can be written as

$$\breve{f}(\breve{p}|p) = \begin{cases} 0 & \text{if } \breve{p} < p \\ \frac{F_{(2)}(x(p)) - F_{(1)}(x(p))}{1 - F_{(1)}(x(p))} \Delta(\breve{p} - p) & \text{if } \breve{p} = p \\ \frac{f_{(2)}(x(\breve{p},p)) \frac{1}{1-\lambda}}{1 - F_{(1)}(x(p))} & \text{if } \breve{p} > p \end{cases}$$

Let $\boldsymbol{X}_i = (t_i, p_i, S_i, \breve{p}_i)$ be an observation. Then the likelihood function $l(\boldsymbol{X}_i|\boldsymbol{\theta})$ in the absence of unobservable heterogeneity is

$$l(\boldsymbol{X}_i|\boldsymbol{\theta}) = h_{tpS}(t, p, S) h_{\breve{p}}(\breve{p}|p, S),$$

where $\boldsymbol{\theta}$ is the vector of parameters determining the shape of h_{tpS} and $h_{\breve{p}S}$. Observe that the private values model is nested as the special case of the above with $\lambda = 0$ (which implies $x(\breve{p}, p) = \breve{p}$).

Unobservable Heterogeneity To account for unobservable heterogeneity, we now extend the setup by distinguishing between the (true) transaction price $\breve{p}_i$ and the (true) reserve price p_i on the one hand and, as in the private values model, between the observed transaction price of object i, which we denote $\hat{\breve{P}}_i$, and the *observed listing price*, which we denote $\hat{P}_i$, on the other hand. Like in the private values model, we stipulate that there is a multiplicative quality index ϑ_i for every object i, which is observed by all market participants but not directly observable by the econometrician. The econometrician only observes a proxy $\hat{\vartheta}_i$, given as

$$\hat{\vartheta}_i = \frac{P_q^{\text{index}}}{P_{q'}^{\text{index}}} \hat{P}_{iq'},$$

where q' and q denote two different quarters and P_q^{index} is the price index in quarter q. The relationship between ϑ_i and $\hat{\vartheta}_i$ is given by

$$\hat{\vartheta}_i = \epsilon_i^Q \vartheta_i,$$

where ϵ_i^Q is the unobserved heterogeneity in quality.

In addition, in the model with a common value component there is the information c about the true objective quality of the object that a priori is only known by the seller but is assumed to be correctly inferred by any bidder in the separating equilibrium of the second-price auction with reserve price signaling. From the perspective of bidders, it seems natural to refer to ϑ as the *observed quality* and to c as the *inferred quality* of an object (as mentioned, we observe neither directly). The willingness to pay for an object of observed quality ϑ and inferred quality c for a buyer of type x is

$$w(x, c, \vartheta) := v(x, c)\vartheta,$$

where $v(x, c) = \lambda c + (1 - \lambda)x$ is as defined in (32).

If the true transaction price $\breve{p}_i$ for object i is determined by an indifferent buyer of type x, as is the case in a second-price auction when the transaction occurs at a price above the reserve, we have

$$\breve{p}_i = v(x, c_i),$$

where $c_i = P_\lambda^{-1}(p_i)$ is the inferred type of the seller of object i who sets the true reserve p_i.

Let $\breve{P}_i$ be the *observed quality-adjusted transaction price.* This is the price observed by the econometrician. It satisfies

$$\breve{P}_i = \frac{\hat{\breve{P}}_i}{\hat{\vartheta}_i} = \breve{p}_i \epsilon_i^Q \quad \text{or equivalently} \quad \breve{p}_i = \frac{\breve{P}_i}{\epsilon_i^Q}.$$

Let p_i be the "true" *reserve price* for object i, with "true" meaning that the reserve satisfies $p_i \vartheta_i$. We assume that p_i is observed without errors by the market participants, but observed with a "discount" noise ϵ_i^D by the econometrician, satisfying the relationship

$$\hat{P}_i = \vartheta_i p_i \epsilon_i^D,$$

where $\hat{P}_i$ is the *listing price* the econometrician observes. The listing price P_i constructed by the econometrician who also observes the quality-proxy $\hat{\vartheta}_i$ is therefore such that

$$P_i = \frac{\hat{P}_i}{\hat{\vartheta}_i} = p_i \epsilon_i^D \epsilon_i^Q.$$

Letting $\epsilon_i^P := \epsilon_i^D \epsilon_i^Q$, we thus have

$$p_i = \frac{P_i}{\epsilon_i^P},$$

and a typical observation is given by

$$\boldsymbol{X}_i = (T_i, P_i, S_i, \breve{P}_i).$$

The likelihood function that accounts for unobservable heterogeneity is then given as

$$l(\boldsymbol{X}_i|\boldsymbol{\theta}) = \sum_{k=1}^{T_i/\tau} \int_0^\infty \int_0^\infty h_{tpS}(T_i - k\tau, P_i/\epsilon^P, S_i) h_{\breve{p}}(\breve{P}_i/\epsilon^Q | P_i/\epsilon^P, S_i) h_t(k\tau) h_Q(\epsilon^Q) h_D(\epsilon^D) d\epsilon^Q d\epsilon^D, \tag{42}$$

where $\breve{P}_i$ is the constructed transaction price and $h_j(\epsilon^j)$ is the density of the error term ϵ^j with $j \in \{Q, D\}$ and where k is the summation variable and τ represents the error term ϵ^T. Rewriting (42) in terms of ϵ^D, ϵ^P using $\epsilon^Q = \epsilon^P/\epsilon^D$ we get

$$l(\boldsymbol{X}_i|\boldsymbol{\theta}) = \sum_{k=1}^{T_i/\tau} \int_0^\infty \int_0^\infty h_{tpS}(T_i - k\tau, P_i/\epsilon^P, S_i) h_{\breve{p}}(\breve{P}_i/(\epsilon^P/\epsilon^D) | P_i/\epsilon^P, S_i) h_t(k\tau) h_P(\epsilon^P) h_D(\epsilon^D) d\epsilon^D d\epsilon^P, \tag{43}$$

where $h_P(\epsilon^P)$ is the density of ϵ^P.

E.5 Estimation

The following describes the general estimation procedure for the model with a common value component. With the exception of one simplification that we impose for the purpose of speeding up computations and that is spelled out at the end of this subsection, this is how we estimated the model when allowing for a positive common-value component λ.

Define

$$y_B := \frac{2x - \underline{x} - \overline{x}}{\overline{x} - \underline{x}} \quad \text{and} \quad y_S := \frac{2c - \underline{c} - \overline{c}}{\overline{c} - \underline{c}},$$

as the valuation and cost normalized to the range $[-1, 1]$ used for Chebyshev polynomials. Virtual valuation and virtual cost functions are parameterized with Chebyshev polynomials:

$$\Phi(x) = \sum_{i=0}^{N} \phi_i \mathcal{T}_i(y_B) \quad \text{and} \quad \Gamma(c) = \sum_{i=0}^{N} \gamma_i \mathcal{T}_i(y_S) \tag{44}$$

where $\mathcal{T}_i(x)$ is the degree i Chebyshev polynomial and N the degree of polynomial approximation.[100]

We impose the restriction $\underline{x} = \underline{c}$ and $\overline{x} = \overline{c}$ and only use the parameters $\underline{c}$ and $\overline{x}$ in the following. (Note that for IPV model in the main text we simply use v instead of x and have $\underline{v} = \underline{x}$, $\overline{v} = \overline{x}$.) δ is the discount factor. The error in the time on market ϵ^T follows a geometric distribution with survival rate β_T. The reserve price noise ϵ^P and the discount noise ϵ^D are lognormally distributed with $\ln \epsilon^P \sim N(\mu_D, \sigma_P)$ and $\ln \epsilon^D \sim N(\mu_D, \sigma_D)$. (The underlying assumption is that the log of unobservable heterogeneity $\ln \epsilon^Q \sim N(0, \sigma_Q)$ and that $\ln \epsilon^P = \ln \epsilon^Q + \ln \epsilon^D$. We will use (ϵ^P, ϵ^D) rather than (ϵ^Q, ϵ^D) in our estimation. We have the restriction $\sigma_P > \sigma_D$.)

The parameters of the empirical model are thus $\{\phi_i\}_{i=0}^N$, $\{\gamma_i\}_{i=0}^N$, β_T, σ_p, δ, $\underline{v}$, $\overline{v}$, $\underline{c}$, $\overline{c}$, ξ, μ_D, σ_D, and λ.

F and G are obtained from their closed-form solutions given the parametrization of Φ and Γ as Chebyshev polynomials. The distributions $F_{(1)}$, $F_{(2)}$ and F_∞, their corresponding densities and $\tilde{\Phi}$ are then given in exactly the same ways as in the private values model.

To satisfy the constraints $\Phi(\overline{x}) = \overline{x}$ and $\Gamma(\underline{c}) = \underline{c}$, we set the parameters ϕ_0 and γ_0 such that

$$\phi_0 = \overline{x} - \sum_{i=1}^{N} \phi_i, \qquad \gamma_0 = \underline{c} - \sum_{i=1}^{N} \gamma_i (-1)^i \tag{45}$$

To see that this is correct, observe that $x = \overline{x}$ corresponds to $y_B = 1$ and $c = \underline{c}$ corresponds to $y_S = -1$ and that $\mathcal{T}_i(1) = 1$ and $\mathcal{T}_i(-1) = (-1)^i$ for all i.

The likelihood function $l(\boldsymbol{X}|\boldsymbol{\theta})$ used for the estimation, given observations $\boldsymbol{X} = (\boldsymbol{X}_i)_{i=1}^n$, is then given as

$$l(\boldsymbol{X}|\boldsymbol{\theta}) = \prod_{i=1}^{n} l(\boldsymbol{X}_i|\boldsymbol{\theta}),$$

where the $l(\boldsymbol{X}_i|\boldsymbol{\theta})$'s are given by (43) and where

$$\boldsymbol{\theta} = (\{\phi_i\}_{i=0}^N, \{\gamma_i\}_{i=0}^N, \beta_T, \sigma_p, \delta, \underline{v}, \overline{v}, \underline{c}, \overline{c}, \xi, \mu_D, \sigma_D, \lambda)$$

[100] The sequence of Chebyshev polynomials starts with $\{\mathcal{T}_i(y)\}_{i=0} = \{1, y, 2y^2 - 1, 4y^3 - 3y, 4y^4 - 8y^2 + 1, ...\}$

is the vector of parameters over which we compute Bayesian posterior expectations of $l(\boldsymbol{X}|\boldsymbol{\theta})$. As in the main text, we assume an uninformative Bayesian prior, so that the Bayesian posterior belief is proportional to the likelihood function.

For the estimation results in this appendix, our specific choice of polynomial terms for the Chebyshev polynomials is two (i.e. $N = 1$). This means that we impose Pareto distributions when we allow for a positive common-value component λ.

To avoid the double complexity of estimating a common value component model and non-linear virtual type functions (i.e. non-Generalized Pareto distributions), we restrict attention to linear virtual type functions. This is sufficient for the purpose of this Section in the Appendix of estimating the weight of the common value component λ. Of course it would not be sufficient for the purpose of estimating how good an approximation of G a mirrored Generalized Pareto distribution is.

The results are displayed in Table 9. For each of the three years, we find $\lambda \approx 0.06$. This means that if the seller's cost increases by $1,000, the gains from trade decrease by $940 and every buyer's willingness to pay increases by $60. In other words, the idiosyncratic component that reduces gains from trade seems to clearly dominate the common value component. Although λ is not exactly 0, it seems small enough to be abstracted away from in our analysis in the main text. There are two possible and plausible forces at play that make the common value component small in the market we study. First, our data set contains only condominium properties. This submarket is more homogenous than the real-estate market as a whole, which plausibly renders common value concerns less important. Second, this is a market with voluntary participation, which can only function well if the adverse selection problem is not too severe.[101]

F Extreme Value Theory

Extreme Value Theory For the convenience of the reader, this appendix provides a summary of the results of the theory of exceedences in extreme value theory that are

[101] This also provides an explanation for the different results obtained by Niedermayer, Shneyerov, and Xu (2015), who study foreclosure real-estate auctions, which do not exhibit voluntary participation.

Estimated Parameter Values						
Parameters	1993		1994		1995	
ϕ_1	0.981	(0.0434)	1.04	(0.0570)	0.962	(0.0331)
γ_1	1.25	(0.0344)	1.24	(0.0306)	1.17	(0.0240)
$\underline{x}$	0.0149	(0.0143)	0.0144	(0.0141)	0.0103	(0.0101)
$\overline{x}$	1.29	(0.0255)	1.28	(0.0224)	1.20	(0.0148)
δ	0.947	(0.0127)	0.948	(0.0191)	0.984	(0.00114)
ξ	0.0645	(0.0841)	0.158	(0.169)	0.0647	(0.0581)
λ	0.0631	(0.0223)	0.0619	(0.0183)	0.0624	(0.0142)
μ_D	0.0980	(0.00505)	0.0843	(0.00340)	0.0735	(0.00235)
σ_D	0.103	(0.00353)	0.0740	(0.00237)	0.0572	(0.00167)
σ_P	0.260	(0.00792)	0.233	(0.00659)	0.230	(0.00638)
β_T	0.0718	(0.0196)	0.101	(0.0480)	0.380	(0.0563)
# Observations	727		830		787	

Table 9: Estimated Parameter Values for 1993 to 1995 with a Common Value Component. Table entries read: Mean (Standard Deviation).

the most important ones for the purposes of our paper. This summary is the content of Theorem 1 below. The theorem says that for any F that satisfies some weak regularity condition,

$$\lim_{u\to 0} 1 - \frac{1 - F(\overline{v} - u(\overline{v} - v))}{1 - F(\overline{v} - u(\overline{v} - \underline{v}))} = 1 - \left(\frac{\overline{v} - v}{\overline{v} - \underline{v}}\right)^{\beta} =: F^*(v), \tag{46}$$

where convergence is uniform and β is some constant. The left-hand side of (46) is the rescaled distribution conditional on being above the threshold $\overline{v} - u(\overline{v} - \underline{v})$. According to Theorem 1, this truncated and rescaled distribution converges to a Generalized Pareto distribution F^* as the threshold $\overline{v} - u(\overline{v} - \underline{v})$ goes to the finite upper bound $\overline{v}$.

The motivation for this theory was the empirical regularity found in many situations that the upper tail of a distribution is well approximated by a (Generalized) Pareto distribution. A prominent example is the distribution of the highest 20 percent of income and wealth in many countries, which was first observed by Vilfredo Pareto.[102] The theory of exceedences within extreme value theory deals with the distribution of a random variable conditional on being above a high threshold (for the original articles see Balkema and De Haan (1974), Pickands (1975); for a textbook see Falk, Hüsler, and Reiss (2010)).

The general principle is described by the Pickands-Balkema-de Haan theorem (also called the second theorem of extreme value theory). For expositional simplicity, we provide a simplified version of the theorem, which is sufficient for our purposes. See Pickands (1975, Theorem 7) and Balkema and De Haan (1974) for the theorem itself. The theorem establishes a connection between the behavior of the maximum of a distribution and its upper tail. The relevant concept for the maximum is the domain of attraction:

Definition 1. *A distribution F is in the* domain of attraction of an extreme value distribution *if there exists a sequence of constants $a_n > 0$ and b_n real for $n = 1, 2, ...$, such that*

$$\lim_{n\to\infty} [F(a_n x + b_n)]^n = F_{max}(x)$$

[102]Other examples include the distribution of the strength of earthquakes in historical data (which tend to contain only the most severe earthquakes); and for the discrete type variant of the Pareto distribution – Zipf's law – the distribution of the frequency of the most common words in a larger text and the sizes of the largest cities in most countries.

for every continuity point x of F_{max} for some non-degenerate distribution function F_{max} (see De Haan and Ferreira, 2006, p. 4).

This means that for n independently and identically distributed random variables, $(\max\{X_1, X_2, ..., X_n\} - b_n)/a_n$ has a non-degenerate distribution as n goes to infinity.

The following theorem holds.

Theorem 1. *(Simplified version of the Pickands-Balkema-de Haan Theorem) Assume F has a finite upper bound and $f(v) > 0$ for all $v \in (\underline{v}, \overline{v})$. Then F has a Generalized Pareto upper tail, formally*

$$\lim_{u \to 0} 1 - \frac{1 - F(\overline{v} - u(\overline{v} - v))}{1 - F(\overline{v} - u(\overline{v} - \underline{v}))} = 1 - \left(\frac{\overline{v} - v}{\overline{v} - \underline{v}}\right)^{\beta}, \tag{47}$$

for some constant β, where convergence is uniform, if and only if F is in the domain of attraction of an extreme value distribution.

The left-hand side of (47) is the rescaled distribution conditional on being above the threshold $\overline{v} - u(\overline{v} - \underline{v})$. The right-hand side is the cumulative distribution function of a finite upper bound Generalized Pareto distribution.

Proof of Theorem 1. See Theorem 7 in Pickands (1975). Note that for our setup ($\overline{v}$ finite and $f(v) > 0$ for all $v \in (\underline{v}, \overline{v})$) the definition of F having a Generalized Pareto upper tail given in Definition 4 in Pickands (1975) simplifies to (47). □

The literature on extreme value theory states several sufficient conditions for a distribution to be in the domain of attraction of an extreme value distribution. We state the one most suitable for our purposes.

Theorem 2. *Assume F has a finite upper bound. F is in the domain of attraction of an extreme value distribution if the von Mises condition*

$$\lim_{v \to \overline{v}} \frac{d}{dv} \left[\frac{1 - F(v)}{f(v)}\right] = \overline{\beta}, \tag{48}$$

for some constant $\overline{\beta}$, holds.

Proof. See, for example, Theorem 1.1.8 in De Haan and Ferreira (2006, p. 15). □

As stated in the literature, even this sufficient condition is weak and is satisfied by all "textbook" continuous distributions, such as uniform, Beta, bounded Generalized Pareto, inverse Weibull and (for the infinite upper bound counterpart of the condition) the normal, exponential, Cauchy, and infinite upper bound Generalized Pareto distribution.

Often, the Generalized Pareto distribution is defined with the parametrization

$$F^*(v) = 1 - \left(1 + \frac{\xi(v-\mu)}{\sigma}\right)^{-1/\xi}.$$

For $\xi < 0$ the distribution has a finite upper bound and corresponds to the parametrization used in this paper with $\underline{v} = \mu$, $\overline{v} = \mu - \sigma/\xi$, and $\beta = -1/\xi$. For $\xi \geq 0$, it has an infinite upper bound and lower bound μ. One obtains the exponential distribution as a special case as $\lim_{\xi \to 0} F^*(v) = 1 - e^{-(v-\mu)/\sigma}$. For $\xi > 0$ and $\sigma = \mu\xi$ one obtains the classical Type I Pareto distribution $F(v) = 1 - (\mu/v)^{1/\xi}$. For $\xi > 0$ one obtains the Type II Pareto distribution.

For infinite upper bounds, convergence can be stated as

$$\left(1 - \frac{1 - F(u+x)}{1 - F(u)}\right) - F_u^*(x) \overset{u\to\infty}{\to} 0,$$

for some Generalized Pareto distribution F_u^*. See the above mentioned references for more details.

Note that the characteristic property of Generalized Pareto distributions is that the inverse hazard rate is linear: $[(1-F(v))/f(v)]' = \xi$. The special cases can be seen as the inverse hazard rate decreasing (bounded Generalized Pareto distribution), constant (exponential distribution), and increasing ((Non-Generalized) Pareto distribution). $\xi < 0$ corresponds to the common monotone hazard rate condition (that is, $f(v)/(1-F(v))$ is increasing). $\xi < 1$ corresponds to Myerson's regularity condition $\Phi'(v) > 0$ and is also necessary to ensure that the distribution has a finite mean.